2. *Admissible Solutions of Higher Order Differential Equations*

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1. Introduction. We use here standard notations in Nevanlinna theory [3], [5].

Let $f(z)$ be a meromorphic function. As usual, $m(r, f)$, $N(r, f)$, and $T(r, f)$ denote the proximity function, the counting function, and the characteristic function of $f(z)$, respectively. Let $\bar{N}(r, f)$ be the counting function for distinct poles of $f(z)$.

A function $\varphi(r)$, $0 \leq r < \infty$, is said to *be* $S(r, f)$ if there is a set $E \subset \boldsymbol{R}^+$ of finite linear measure such that $\varphi(r) = o(T(r, f))$ as $r \to \infty$, $r \notin E$. A meromorphic function $a(z)$ is said to be *small with respect to* $f(z)$ if $T(r, a) = S(r, f)$. Let $a_j(z)$, $j = 1, \cdots, n$, be meromorphic functions. A function $w(z)$ is *admissible with respect to* $a_j(z)$, if $T(r, a_j) = S(r, w)$, $j = 1, \cdots, n$.

For a differential monomial $M[w] = a(z)w^{n_0}(w')^{n_1} \cdots (w^{(m)})^{n_m}$ in w, we put $\gamma_M = n_0 + n_1 + \cdots + n_m$ and $\Gamma_M^\mu = \mu n_0 + (\mu+1)n_1 + \cdots + (\mu+m)n_m$, and call *degree* and *weight-*μ of $M[w]$, respectively. We write Γ_M^1 simply as Γ_M. Let $\Omega(z)$ be a differential polynomial with meromorphic coefficients:

$$\Omega[w] = \sum_{\lambda \in I} M_\lambda[w] = \sum_{\lambda \in I} a_\lambda(z) w^{n_0}(w')^{n_1} \cdots (w^{(m)})^{n_m},$$

where $a_\lambda(z)$ are meromorphic functions, I is a finite set of multi-indices $\lambda = (n_0, n_1, \cdots, n_m)$. We define *degree* γ_Ω and *weight-*μ Γ_Ω^μ of Ω by $\gamma_\Omega = \max_{\lambda \in I} \gamma_{M_\lambda}$ and $\Gamma_\Omega^\mu = \max_{\lambda \in I} \Gamma_{M_\lambda}^\mu$, respectively.

A meromorphic solution $w(z)$ of the differential equation $\Omega[w] = 0$ is *admissible solution*, if $w(z)$ is admissible w.r.t. $a_\lambda(z)$, $\lambda \in I$.

$\Omega[w]$ is said to *satisfy the condition* (GL) *if, for any* $\mu \geq 1$,

(GL) *there is an index* i_μ *such that* $\Gamma_{M_{i_\mu}}^\mu > \Gamma_{M_i}^\mu$ *if* $i \neq i_\mu$.

This condition (GL) is due to Gackstatter-Laine [2], who investigated the equation

$$w'^n = \sum_{j=0}^{m} a_j(z) w^j \qquad (0 \leq m \leq 2n), \tag{1.1}$$

and conjectured that it would not admit any admissible solution if $1 \leq m \leq n-1$. In this respect, Toda [7] proved the following theorem.

Theorem A. *The differential equation* (1.1) *does not possess any admissible solutions if* $1 \leq m \leq n-1$, *except for the case when* $n-m$ *is a divisor of* n *and* (1.1) *is of the following form*:

$$w'^n = a_m(z)(w+\alpha)^m, \qquad \textit{where } \alpha \textit{ is a constant.}$$

Recently, Toda [8] studied more general differential equation