94. On the Automorphism Groups of Edge-coloured Digraphs

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1. **Introduction.** For any finite group $G=\{g_1, g_2, \cdots, g_q\}$, we construct an edge-coloured strongly connected digraph $\Delta=\Delta(G)$ with the vertex-set $V\Delta=\{g_1, g_2, \cdots, g_q\}$ such that for any two vertices u and v of Δ both (u, v) and (v, u) are directed edges of Δ and they are coloured with colours $u^{-1}v$ and $v^{-1}u$ respectively. Then the (colour-preserving) automorphism group $\mathrm{Aut}\,\Delta$ of Δ on $V\Delta$ is isomorphic to the regular representation [4] of G as a permutation group ([5, p. 96, Lemma 3.1]). On the other hand, Frucht [1] and Sabidussi [2] proved the following: For any finite group G and any integer $k\geqq 3$ there exist infinitely many connected k-regular (undirected) graphs Γ in which $V\Gamma$ has a disjoint union decomposition $V\Gamma=\sum_{i=1}^{q} V_i$ ($q=|G|$) such that the automorphism group $\mathrm{Aut}\,\Gamma$ of Γ acts faithfully on the set $\{V_1, V_2, \cdots, V_q\}$ by the natural action and the permutation group derived by its action is isomorphic to the regular representation of G as a permutation group.

In this peper we shall extend the above. Let Δ be an edge-coloured digraph and C be the set of colours c with which at least one directed edge of Δ is coloured. We define a uniquely definite positive integer $\lambda(\Delta)$ as follows. For any vertex x of Δ and c in C let $\lambda_{\mathrm{in}}(x; c)$ denote the number of directed edges with colour c having x as head and $\lambda_{\mathrm{out}}(x; c)$ denote the number of directed edges with colour c having x as tail. We define $\lambda_{\max}(\Delta)=\max\{\lambda_{\mathrm{in}}(x; c), \lambda_{\mathrm{out}}(x; c) : x\in V\Delta, c\in C\}$ and $\lambda(\Delta)=\max\{\lambda_{\max}(\Delta)+1, 3\}$. The purpose of this paper is to prove

Theorem. *Let Δ be an edge-coloured weakly connected digraph with $|V\Delta|=n$. Then for any integer $k\geqq\lambda(\Delta)$ there exist infinitely many connected k-regular (undirected) graphs Γ in which $V\Gamma$ has a disjoint union $V\Gamma=\sum_{i=1}^{n} V_i$ such that $\mathrm{Aut}\,\Gamma$ acts faithfully on the set $\{V_1, V_2, \cdots, V_n\}$ by the natural action and the permutation group derived by its action is isomorphic to the (colour-preserving) automorphism group $\mathrm{Aut}\,\Delta$ of Δ on $V\Delta$ as a permutation group.*

2. **Preliminaries.** Unless stated otherwise, all graphs are finite, undirected, simple and loopless. If an edge e joins two vertices u and v, we write $e=[u, v]=[v, u]$. If $\mathrm{Aut}\,\Gamma=1$, Γ is called asymmetric.

Now we introduce a notion of the type [1] $(a_1, a_2, \cdots, a_r)$ $(r=m(m-1)/2)$ of a vertex v of valency m in a graph Γ. Let $u_1, u_2, \cdots, u_m$ be the adjacent vertices of v. We define the number α_{ij} $(i<j)$ as follows:

α_{ij} = the minimum length of circuits which contain the two edges $[u_i, v]$