84. *Regular Elements of Abstract Affine Near-rings*

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1. Introduction. In his paper [2], Steinfeld characterizes the regular elements of a ring in terms of quasi-ideals.

The purpose of this note is to extend the above result to a class of abstract affine near-rings. An example is given to show that the result does not hold for arbitrary near-rings.

For the basic terminology and notation we refer to [1].

2. Preliminaries. Let N be a near-ring, which always means right one throughout this note.

If A, B and C are three non-empty subsets of N, then AB (ABC) denotes the set of all finite sums of the form $\sum a_k b_k$ with $a_k \in A$, $b_k \in B$ ($\sum a_k b_k c_k$ with $a_k \in A$, $b_k \in B$, $c_k \in C$), and $A * B$ denotes the set of all finite sums of the form $\sum (a_k(a'_k + b_k) - a_k a'_k)$ with a_k, $a'_k \in A$, $b_k \in B$. Note that $ABC = (AB)C \subseteq A(BC)$ in general, and that $ABC = (AB)C = A(BC)$ if $A \subseteq N_d$, where N_d is the set of all distributive elements of N.

A *right N-subgroup* (*left N-subgroup*) of N is a subgroup H of $(N, +)$ such that $HN \subseteq H$ ($NH \subseteq H$). A *quasi-ideal* of N is a subgroup Q of $(N, +)$ such that $QN \cap NQ \cap N * Q \subseteq Q$. Right N-subgroups and left N-subgroups are quasi-ideals. The intersection of a family of quasi-ideals is again a quasi-ideal.

Lemma 1. *Let e be an idempotent element of a near-ring N, and let R be a right N-subgroup of N. Then the following assertions hold*:

(i) $R(Ne) = Re$.

(ii) *Re is a quasi-ideal of N such that* $Re = R \cap Ne$.

Proof. (i) We have $R(Ne) = RNe \subseteq Re$ and $Re = Ree \subseteq RNe = R(Ne)$. So $R(Ne) = Re$.

(ii) Since R and Ne are quasi-ideals of N, it suffices to prove the relation $Re = R \cap Ne$. As $Re \subseteq R \cap Ne$, we have to show only $R \cap Ne \subseteq Re$. Any element x of $R \cap Ne$ has the form $x = r = ne$ with $r \in R$, $n \in N$, whence $x = ne = nee = re \in Re$.

For an element n of a near-ring N, $(n)_r$ ($(n)_l$) denotes the right (left) N-subgroup of N generated by n, and $[n]$ denotes the subgroup of $(N, +)$ generated by n.

An element n of a near-ring N is called *regular* if $n = nxn$ for some element x of N.