80. *Properties of Certain Integral Operator*

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1. Introduction. Let $\mathcal{A}_n$ denote the class of functions of the form

(1.1) $$f(z)=z+\sum_{k=n+1}^{\infty} a_k z^k \qquad (n \in \mathcal{N}=\{1,2,3,\cdots\})$$

which are analytic in the unit disk $\mathcal{U}=\{z:|z|<1\}$.

A function $f(z)$ in the class $\mathcal{A}_n$ is said to be a member of the class $\mathcal{A}_n(\alpha)$ if it satisfies

(1.2) $$\left|\frac{f(z)}{z}-1\right|<1-\alpha \qquad (z \in \mathcal{U})$$

for some α $(0 \leqq \alpha < 1)$.

Let the functions $f(z)$ and $g(z)$ be analytic in the unit disk $\mathcal{U}$. Then the function $f(z)$ is said to be subordinate to $g(z)$ if there exists a function $w(z)$ analytic in $\mathcal{U}$, with $w(0)=0$ and $|w(z)|<1$ $(z \in \mathcal{U})$, such that

(1.3) $$f(z)=g(w(z)) \qquad (z \in \mathcal{U}).$$

We denote this subordination by

(1.4) $$f(z) \prec g(z).$$

In particular, if $g(z)$ is univalent in $\mathcal{U}$, then the subordination (1.4) is equivalent to $f(0)=g(0)$ and $f(\mathcal{U}) \subset g(\mathcal{U})$ (cf. [2]).

This concept of subordination can be traced to Lindelöf [5], but Littlewood ([6], [7]) and Rogosinski ([10], [11]) introduced the term and discovered the basic properties.

For a function $f(z)$ belonging to the class $\mathcal{A}_n$, we define the generalized Libera integral operator J_c by

(1.5) $$J_c(f(z))=\frac{c+1}{z^c}\int_0^z t^{c-1} f(t)\,dt \qquad (c \geqq 0).$$

The operator J_c, when $c \in \mathcal{N}$, was introduced by Bernardi [1]. In particular, the operator J_1 was studied earlier by Libera [4] and Livingston [8].

2. Properties of the operator J_c. In order to derive our results, we have to recall here the following lemma due to Miller and Mocanu [9] (also Jack [3]).

Lemma. *Let the function*

(2.1) $$w(z)=b_n z^n + b_{n+1} z^{n+1} + \cdots \qquad (n \in \mathcal{N})$$

be regular in the unit disk $\mathcal{U}$ *with* $w(z) \not\equiv 0$ $(z \in \mathcal{U})$. *If* $z_0=r_0 e^{i\theta_0}$ $(r_0<1)$ *and*

(2.2) $$|w(z_0)|=\max_{|z| \leqq r_0} |w(z)|,$$

then

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