79. *Negativity and Vanishing of Microfunction Solution Sheaves at the Boundary*

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Introduction. Let M be a real analytic manifold with a complexification X. Let V be a $\mathbb{C}^\times$-conic involutive submanifold of $\mathring{T}^*X(=T^*X\setminus X)$, and let $\mathfrak{M}$ be a coherent $\mathcal{E}_X$-module with constant multiplicity along V. Moreover let Ω be an open subset of M with real analytic boundary $N=\partial\Omega$. The aim of this note is to give vanishing theorems for the cohomology groups of the complex $\boldsymbol{R}\,\underline{\mathrm{Hom}}_{\mathcal{E}_X}(\mathfrak{M}, \mathcal{C}_{\Omega|X})$ where $\mathcal{C}_{\Omega|X}$ is the complex of microfunctions at the boundary introduced by P. Schapira [8] (see § 1.1 for the definition).

The vanishing of the complex $\boldsymbol{R}\,\underline{\mathrm{Hom}}_{\mathcal{E}_X}(\mathfrak{M}, \mathcal{C}_M)$ has been studied by M. Sato *et al.* [6], M. Kashiwara [3] and Kashiwara-Schapira [5], and we study in this note an analogous problem at the boundary.

1. Preliminary and a lemma. 1.1. Let M be a real analytic manifold of dimension n with a complexification X, and let Ω be an open subset of M with real analytic boundary $N=\partial\Omega$.

The cotangent bundle T^*X of X is endowed with the sheaf $\mathcal{E}_X$ of microdifferential operators of finite order. Refer to M. Sato *et al.* [6] and P. Schapira [7] for detailed account of $\mathcal{E}_X$. Let $T^*_\Omega X$ denote the micro-support of $\mathbb{Z}_\Omega$ due to [4], and let $\mathcal{C}_{\Omega|X}$ be the complex of microfunctions along $T^*_\Omega X$ introduced by P. Schapira [8]. With the bifunctor $\mu\mathrm{hom}(\cdot,\cdot)$ constructed by Kashiwara-Schapira [4], the complex $\mathcal{C}_{\Omega}|_X$ is explicitly given by

$$\mathcal{C}_{\Omega|X}=\mu\mathrm{hom}(\mathbb{Z}_\Omega, \mathcal{O}_X)\otimes or_M[n]$$

where or_M denotes the orientation sheaf on M.

1.2. We follow the notation in § 1.1. Let V be a $\mathbb{C}^\times$-conic involutive submanifold of $\mathring{T}^*X$. We recall the Levi form $\mathcal{L}_\Lambda(V)(p)$ of V along $\Lambda=T^*_M X$ at $p\in\Lambda\cap V$. Take a system of functions $(f_1,\cdots,f_l)$ so that $V=\{q\in\mathring{T}^*X;\ f_1(q)=\cdots=f_l(q)=0\}$ locally in a neighborhood p. Then $\mathcal{L}_\Lambda(V)(p)$ denotes the Hermitian form given by the matrix $(\{f_i, f_j^c\})_{1\le i,j\le l}$. Here f_j^c is the complex conjugate of f_j and $\{\cdot,\cdot\}$ is the Poisson bracket. We remark that the signature of $\mathcal{L}_\Lambda(V)(p)$ is independent of the choice of $(f_1,\cdots,f_l)$. Refer to M. Sato *et al.* [6] and Kashiwara-Schapira [5].

1.3. Let X be a C^∞ manifold. Then $D^b(X)$ denotes the derived category of the category of bounded complexes of sheaves on X. For $F\in Ob(D^b(X))$, $\mathrm{SS}(F)$ is its micro-support. Let Z_1 and Z_2 be two subsets in X. Then $C(Z_1,Z_2)$ is the tangent cone for the pair (Z_1,Z_2). Refer to Kashiwara-Schapira [4] for all in this § 1.3.