70. *Fourier Coefficients of Certain Eisenstein Series*

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We fix natural numbers $q\geq 3$, k, $n\geq 1$ once and for all. For $\gamma, \delta \in M_n(\boldsymbol{Z})$, we write $(\gamma, \delta)=1$ if $(\gamma\delta)$ is a lower $n\times 2n$ submatrix of some element of $Sp(n;\boldsymbol{Z})$, and put $H_n := \{z \in M_n(\boldsymbol{C}) \mid {}^tz=z, \operatorname{Im} z>0\}$. We fix such a pair γ, δ hereafter. We consider Eisenstein series

$$E(z, s, k; (\gamma, \delta)) := \sum \det(cz+d)^{-k} \operatorname{abs}(\det(cz+d))^{-2s} \quad (z \in H_n,\ s \in \boldsymbol{C}),$$

where (c, d) runs over $G_n(q)\backslash\{(c, d) \mid (c, d)=1,\ c\equiv\gamma,\ d\equiv\delta \bmod q\}$ and $G_n(q)=\{a \in GL_n(\boldsymbol{Z}) \mid a\equiv 1_n \bmod q\}$. Our aim is to study Dirichlet series which appear in Fourier coefficients of $E(z, s, k; (\gamma, \delta))$. We denote by $E'(z, s, k; (\gamma, \delta))$ a partial sum of $E(z, s, k; (\gamma, \delta))$ with $\det c\neq 0$. For a ring R, we denote by $\Lambda_n(R)$ the set of all symmetric matrices of degree n with entries in R and put $\Lambda_n := \Lambda_n(\boldsymbol{Z})$. By Λ_n' we denote the set of all half-integral matrices of degree n, i.e. matrices a such that $2a \in \Lambda_n$ and diagonals of a are integers. Following [3], we put, for $z \in H_n$

$$\sum_{a\in\Lambda_n} \det(z+a)^{-\alpha} \det(\bar{z}+a)^{-\beta} = \sum_{h\in\Lambda_n'} e(\operatorname{tr} hx)\xi(y, h; \alpha, \beta),$$

where $x=\operatorname{Re} z$, $y=\operatorname{Im} z$, $e(w)$ means $\exp(2\pi i w)$ and the function ξ is defined by the above and is fully studied in [3]. We have

$$E'(z, s, k; (\gamma, \delta)) = q^{-n(k+2s)} \sum_{h\in\Lambda_n'} \xi(q^{-1}y, h; s+k, s)\zeta(h; k, (\gamma, \delta); s)e(\operatorname{tr} hx/q)$$

where $x=\operatorname{Re} z$, $y=\operatorname{Im} z$ and

$$\zeta(h; k, (\gamma, \delta); s) = \sum_c \sum_d \det(c)^{-k} \operatorname{abs}(\det(c))^{-2s} e(q^{-1} \operatorname{tr} hc^{-1}d).$$

where c runs over $G_n(q)\backslash\{c \in M_n(\boldsymbol{Z}) \mid c\equiv\gamma \bmod q,\ \det c\neq 0\}$ and d runs over $\{d \in M_n(\boldsymbol{Z}) \bmod qc\Lambda_n \mid (c, d)=1,\ d\equiv\delta \bmod q\}$. Decompose q as $q=\prod q_i$ where q_i is a power of a prime p_i and for a Dirichlet character χ defined modulo q, we denote by χ_i a Dirichlet character defined modulo q_i such that $\chi=\prod \chi_i$. Then we have

$$\zeta(h; k, (\gamma, \delta); s) = 2\varphi(q)^{-1} \sum_{\substack{\chi \bmod q \\ \chi(-1)=(-1)^k}} \prod_{p\nmid q} b_p((p^{k+2s}\chi(p))^{-1}, h)$$
$$\times \prod_i b_{p_i}((p_i^{k+2s}(\prod_{j\neq i} \chi_j)(p_i))^{-1}; h, \chi, (\gamma, \delta)),$$

where φ is the Euler's function and we put, for $h \in \Lambda_n'$

$$b_p(x, h) = \sum_{r\in\Lambda_n(\boldsymbol{Q}_p)/\Lambda_n(\boldsymbol{Z}_p)} x^{\operatorname{ord}_p \nu(r)} e(\operatorname{tr} hr),$$

where $\nu(r)$ is the product of reduced denominators of elementary divisors of r. To define the function b_{p_i}, we put, for a power Q of a prime p, $h \in \Lambda_n'$ and a Dirichlet character χ defined modulo Q,

$$B_p(x; h, \chi; (\gamma, \delta), Q) = \sum_{c\in U_n\backslash c(n;p)} x^{\operatorname{ord}_p \det c} \sum_{\substack{d \bmod Qc\Lambda_n \\ c^td\in\Lambda_n}} e(Q^{-1} \operatorname{tr} hc^{-1}d) \sum_g \chi(\det g),$$

where g runs over $GL_n(\boldsymbol{Z}/Q\boldsymbol{Z})$ with $c\equiv g\gamma \bmod Q$ and $d\equiv g\delta \bmod Q$ (as a