8. Minimal Quasi-ideals in Abstract Affine Near-rings

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1. Introduction. In his paper [3], Stewart answered Problem 6.1 in [2] in the affirmative, proving that a quasi-ideal of a ring is minimal if and only if any two of its non-zero elements generate the same left ideal and the same right ideal of the ring.

Our aim is to generalize the above result to a class of abstract affine near-rings. An example is given to show that the result does not hold for arbitrary near-rings.

2. Left ideals, right ideals and quasi-ideals. Let N be a near-ring, which always means right one throughout this note. If A and B are two non-empty subsets of N, then AB denotes the set of all finite sums of the form $\sum a_k b_k$ with $a_k \in A$, $b_k \in B$, and $A * B$ denotes the set of all finite sums of the form $\sum (a_k(a'_k + b_k) - a_k a'_k)$ with $a_k, a'_k \in A$, $b_k \in B$.

A *left ideal* of N is a normal subgroup L of $(N, +)$ such that $N * L \subseteq L$, and a *right ideal* of N is a normal subgroup R of $(N, +)$ such that $RN \subseteq R$. A *quasi-ideal* of N is a subgroup Q of $(N, +)$ such that $N * Q \cap NQ \cap QN \subseteq Q$. A non-zero quasi-ideal Q is *minimal* if the only quasi-ideals of N contained in Q are $\{0\}$ and Q. Left ideals and right ideals are quasi-ideals, and the intersection of a family of quasi-ideals is again a quasi-ideal.

A near-ring N is called an *abstract affine near-ring* if N is abelian and $N_0 = N_d$, where N_0 is the zero-symmetric part of N and N_d is the set of all distributive elements of N.

These definitions lead immediately to

Lemma. *Let N be an abstract affine near-ring.*

(a) *A subgroup L of $(N, +)$ is a left ideal of N if and only if $N_0 L \subseteq L$.*

(b) *If P is a subgroup of $(N, +)$, then $N_0 P$ is a left ideal of N and PN is a right ideal of N.*

(c) *A subgroup Q of $(N, +)$ is a quasi-ideal of N if and only if $N_0 Q \cap QN \subseteq Q$.*

3. Main result. If x is an element of a near-ring N, $(x)_l$ (respectively, $(x)_r$) denotes the left (respectively, right) ideal of N generated by x. Now we are ready to state the main result of this note.

Theorem. *A quasi-ideal Q of an abstract affine near-ring N is minimal if and only if any two of its non-zero elements generate the same left ideal and the same right ideal of N.*

Proof. Suppose that Q is a minimal quasi-ideal of an abstract affine