69. Applications of B(P, α)-refinability for Generalized Collectionwise Normal Spaces

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Introduction. In [5] the authors introduced the notion of $B(P,\alpha)$-refinability and used it to obtain new covering characterizations for normal and collectionwise normal spaces. In this paper we generalize various known results by obtaining analogous characterizations for collectionwise subnormal and strong-collectionwise subnormal spaces.

The properties P considered in this paper will be discrete (D), locally finite (LF), closure-preserving (CP), hereditarily closure-preserving (HCP) and point finite (PF). The symbol α will denote any countable ordinal.

Definition 1. A space X is *$B(P,\alpha)$-refinable* provided every open cover $\mathcal{U}$ of X has a refinement $\mathcal{E}=\bigcup\{\mathcal{E}_\beta:\beta<\alpha\}$ which satisfies i) $\{\bigcup\mathcal{E}_\beta:\beta<\alpha\}$ partitions X, ii) for every $\beta<\alpha$, $\mathcal{E}_\beta$ is a relatively P collection of closed subsets of the subspace $X-\bigcup\{\bigcup\mathcal{E}_\mu:\mu<\beta\}$, and iii) for every $\beta<\alpha$, $\bigcup\{\mathcal{E}_\mu:\mu<\beta\}$ is a closed set.

The collection $\mathcal{E}$ is often called a *$B(P,\alpha)$-refinement* of $\mathcal{U}$.

Definition 2. Let $\mathcal{G}$ be a collection of open subsets of a space X. We refer to $\mathcal{G}$ as a *θ-collection* (*almost θ-collection*) provided we can write $\mathcal{G}=\bigcup\{\mathcal{G}_n:n\in N\}$ such that for every $x\in X$, there exists $n(x)\in N$ such that $\mathcal{G}_{n(x)}$ is LF (PF) at x.

Definition 3. (1) Let $\mathcal{F}$ be a collection of subsets of a space X. We call $\mathcal{G}=\bigcup\{\mathcal{G}_n:n\in N\}$ an (*almost θ-expansion*) of $\mathcal{F}$ provided (i) $\mathcal{G}_n$ is an open expansion of $\mathcal{F}$ for every $n\in N$, and (ii) $\mathcal{G}$ is an (almost) θ-collection.

(2) A space X is (*almost*) *θ-expandable* provided every LF collection of closed subsets of X has an (almost) θ-expansion.

(3) A space X is (*almost*) *discretely-θ-expandable* provided every discrete collection of closed subsets of X has an (almost) θ-expansion.

Expandable and θ-expandable spaces have been studied in [3, 4, 9, 10].

Definition 4. A space X is collectionwise subnormal (CWSN) provided every discrete collection $\mathcal{D}$ of closed subsets of X has a pairwise disjoint G_δ-expansion which is also an almost θ-expansion of $\mathcal{D}$.

In 1979 Chaber [1] obtained the following result.

Theorem 1. *A space X is subparacompact iff X is CWSN and θ-refinable.*

Here we generalize Chaber's result by using the notion of $B(D,\omega)$-refinability.

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