68. A Remark on $B(P, \alpha)$-refinability

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Introduction. Recently a number of general topological properties have been introduced in order to obtain covering characterizations of generalized normal and paracompact spaces. In particular see [1, 2, 7, 10] for such characterizations of subparacompact, θ-refinable, collectionwise normal and collectionwise subnormal spaces. In this paper we consider the general property of $B(P, \alpha)$-refinable and show how this notion is used to generalize known results for normal and collectionwise normal spaces.

The union of any family $\mathcal{U}$ will be denoted by $\mathcal{U}^*$. The properties P considered in this paper will be discrete (D), locally finite (LF) and closed (C). Countable ordinals will be denoted by λ and α will be any ordinal.

Definition 1. A space X is *$B(P, \alpha)$-refinable* provided every open cover $\mathcal{U}$ of X has a refinement $\mathcal{E}=\cup\{\mathcal{E}_\beta : \beta<\alpha\}$ which satisfies i) $\{\cup\mathcal{E}_\beta : \beta<\alpha\}$ partitions X, ii) for every $\beta<\alpha$, $\mathcal{E}_\beta$ is a relatively P collection of closed subsets of the subspace $X-\cup\{\cup\mathcal{E}_\mu : \mu<\beta\}$, and iii) for every $\beta<\alpha$, $\cup\{\cup\mathcal{E}_\mu : \mu<\beta\}$ is a closed set. For the case $P=C$, we require $\mathcal{E}_\beta$ to be a one-to-one partial refinement of $\mathcal{U}$ for each $\beta<\alpha$.

The collection $\mathcal{E}$ is often called a *$B(P, \alpha)$-refinement* of $\mathcal{U}$.

In [6, 7] the author has used the property of weakly $\bar{\theta}$-refinable to obtain several open cover characterizations for normal and collectionwise normal spaces. The following are modifications of this idea.

Definition 2. An open cover $\mathcal{G}=\cup\{\mathcal{G}_n : n\in N\}$ of a space X is a (k^-) *bded-weak $\bar{\theta}$-cover* if (i) the collection $\{\mathcal{G}_n^* : n\in N\}$ is point finite and (ii) for each n, there exist an integer $k(n)$ ($\leq k$) such that $X=\{x : 0<ord(x, \mathcal{G}_n)\leq k(n), n\in N\}$. Spaces for which each open cover has a refinement with the above property are called *(k^-)-bded-weak $\bar{\theta}$-refinable.*

Remark. A k-bded weak $\bar{\theta}$-cover is equivalent to a boundly weak $\bar{\theta}$-cover, as defined in [10].

Main results.

Theorem 1. *A space X is bded-weak $\bar{\theta}$-refinable iff X is 1-bded weak $\bar{\theta}$-refinable.*

Proof. The sufficiency is clear. Let $\mathcal{G}=\{\mathcal{G}_n : n\in N\}$ be a bded-weak $\bar{\theta}$-cover of X with $k(n)$ defined as above.

For each $x\in X$ and every $n, j\in N$, define $W(n, x)=\cap\{G\in\mathcal{G}_n : x\in G\}$, and $\mathcal{W}(n, j)=\{W(n, x) : ord(x, \mathcal{G}_n)=j\}$ so that if $ord(x, \mathcal{G}_n)=j$, then $ord(x, \mathcal{W}(n, j))=1$. Define $\mathcal{W}=\cup\{\mathcal{W}(n, j) : 0<j\leq k(n), n\in N\}$. It should

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