63. On the Unitarizability of Principal Series Representations of $\mathfrak{p}$-adic Chevalley Groups

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1. In this note, we shall determine the unitarizability of unramified principal series representations of $\mathfrak{p}$-adic Chevalley groups of classical types. Detailed proofs of all the results stated here are given in [7].

2. Let k be a non-archimedean local field, $\mathfrak{O}$ be the maximal compact subring and ϖ be a prime element of k. Set $q=|\mathfrak{O}/\varpi\mathfrak{O}|$. The following theorem is our main tool in this research.

Theorem 1. *Let N be the group of k-rational points of a unipotent algebraic group defined over k. Let T be a distribution of positive type on N. Then, for any $\alpha \in C_c^\infty(N)$, the convolution $T*\alpha$ is a bounded function on N.*

3. Let $\boldsymbol{G}$ be a universal Chevalley group defined over k in the sense of Steinberg [6]. Let $\boldsymbol{T}$ be a maximal k-split torus and $\boldsymbol{B}$ be a Borel subgroup defined over k which contains $\boldsymbol{T}$. Let $\boldsymbol{N}$ be the unipotent radical of $\boldsymbol{B}$. Let G, T, B and N stand for the groups of k-rational points of $\boldsymbol{G}$, $\boldsymbol{T}$, $\boldsymbol{B}$ and $\boldsymbol{N}$ respectively. Let Σ be the root system and $\Delta=\{\alpha_1, \alpha_2, \cdots, \alpha_\ell\}$ be the set of simple roots determined by $(\boldsymbol{G}, \boldsymbol{B}, \boldsymbol{T})$, where ℓ is the rank of $\boldsymbol{G}$. Let Σ^+ be the set of positive roots and W be the Weyl group. For $w \in W$, set $\Psi_w^+=\{\alpha \in \Sigma^+ \mid w\alpha<0\}$. We have $B=TN=NT$ and T (resp. N) is generated by $h_\alpha(t)$ (resp. $x_\alpha(t)$) for $\alpha \in \Sigma^+$, $t \in k^\times$ (resp. $t \in k$) in the notation of [6]. If $\alpha \in \Sigma$, let $\check{\alpha} \in \operatorname{Hom}(\boldsymbol{G}_m, \boldsymbol{T})$ be the co-root of α and set $a_\alpha=\check{\alpha}(\varpi)=h_\alpha(\varpi) \in T$. For $\alpha, \beta \in \Sigma$, we set $\langle\alpha, \beta\rangle=\langle\alpha, \check{\beta}\rangle_1$ with the canonical pairing $\langle\ ,\ \rangle_1$ of a root with a co-root. Let δ_B denote the modular function of B. For a quasi-character χ of T, let $PS(\chi)$ denote the space of all locally constant functions φ on G which satisfy

$$\varphi(tng)=\delta_B(t)^{1/2}\chi(t)\varphi(g) \qquad \text{for any } t \in T,\ n \in N,\ g \in G.$$

Let $\pi(\chi)$ denote the admissible representation of G realized on $PS(\chi)$ by right translations.

Let K be the subgroup of G generated by $x_\alpha(t)$, $\alpha \in \Sigma$, $t \in \mathfrak{O}$. Then K is a maximal compact subgroup of G and we have the Iwasawa decomposition $G=BK$. We call χ *unramified* if χ is trivial on $T \cap K$, the group generated by $h_\alpha(t)$, $\alpha \in \Sigma^+$, $t \in \mathfrak{O}^\times$. Let X be the group of all unramified quasi-characters of T. The map $\chi \rightarrow (\chi(a_{\alpha_1}), \chi(a_{\alpha_2}), \cdots, \chi(a_{\alpha_\ell}))$ defines an isomorphism $X \cong (\boldsymbol{C}^\times)^\ell$ and we consider X as a complex Lie group. We call χ *regular* if $w\chi \neq \chi$ for any $w \in W$, $w \neq 1$. Let X^r (resp. X^i) denote the set of all $\chi \in X$ which are regular (resp. regular and $\pi(\chi)$ is irreducible). Let