61. *Limiting Amplitude Principle for Acoustic Propagators in Perturbed Stratified Fluids*

By Koji KIKUCHI*) and Hideo TAMURA**)

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Introduction. Recently the spectral problems for acoustic operators $L=-c(x)^2\Delta$ in perturbed stratified fluids have been studied by several authors ([1], [2], [7], [8]). Under suitable assumptions on the behavior of sound speed $c(x)$ at infinity, non-existence of eigenvalues and the principle of limiting absorption have been proved for the operator L. In the present note we study the principle of limiting amplitude for L which has not been discussed in detail in the works above.

1. Limiting amplitude principle. The precise formulation of the obtained result requires several notations and assumptions.

We work in the 3-dimensional space R^3_x and write the coordinates in R^3_x as $x=(y, z)$ with $y\in R^1$ and $z=(z_1, z_2)\in R^2$. Let Δ be the 3-dimensional Laplace operator and let $c_0(y)>0$ be the sound speed in the fluid under consideration, which depends on the depth variable y only. In particular, we here are interested in the case where $c_0(y)$ takes the constant values c_-, c_0 and c_+ for $y<0$, $0<y<h$ and $y>h$, respectively. Then the acoustic wave in the stratified fluid is governed by the wave equation $\partial^2u/\partial t^2-c_0(y)^2\Delta u=0$. On the other hand, the acoustic wave in a perturbed stratified fluid which we consider here is also governed by a similar equation $\partial^2u/\partial t^2-c(x)^2\Delta u=0$, where the sound speed $c(x)$ is assumed to satisfy the following assumptions:

(A.1) $\quad 0<c_m\leqq c(x)\leqq c_M \quad$ for some c_m and c_M.

(A.2) $\quad c(x)-c_0(y)=O(|x|^{-\rho}),\quad |x|\to\infty, \quad$ for some $\rho>1$.

We consider the above wave equation in the Hilbert space $L^2(R^3_x; c(x)^{-2}dx)$. Define the acoustic operator L as $L=-c(x)^2\Delta$. Then L is symmetric in this space and it admits a unique selfadjoint realization. We denote by the same notation L this self-adjoint realization and by $R(\zeta; L)$, $\operatorname{Im}\zeta\neq 0$, the resolvent of L; $R(\zeta; L)=(L-\zeta)^{-1}$. As is easily seen, the operator L is positive (zero is not an eigenvalue) and the domain $D(L)$ is given by $D(L)=H^2(R^3_x)$, $H^s(R^3_x)$ being the Sobolev space of order s. We here summarize the spectral properties of L obtained by the works [1], [2], [7] and [8] under assumptions (A.1) and (A.2): (i) L has no eigenvalues; (ii) The boundary values $R(\lambda\pm i0; L)$, $\lambda>0$, of $R(\lambda\pm i\kappa; L)$ as $\kappa\to 0$ exist as an operator from L^2_α into $L^2_{-\alpha}$ for $\alpha>1/2$, where $L^2_\alpha=L^2_\alpha(R^3_x)$ is the weighted L^2

*) Department of Mathematics, Faculty of Liberal Arts, Shizuoka University.
**) Department of Mathematics, Ibaraki University.