7. Iwasawa Theoretical Residue Formulas for Algebraic Tori*)

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(Communicated by Shokichi IYANAGA, M. J. A., Jan. 12, 1989)

Introduction. In his paper [2], J. Coates gave an Iwasawa theoretical analogue of the analytic class number formula. The aim of this note is to give a residue formula for an algebraic torus, which generalizes Coates' formula under a certain condition and can also be regarded as an Iwasawa theoretical analogue of the analytic class number formula for an algebraic torus introduced by T. Ono and J.-M. Shyr ([4], [10]). We must mention that Iwasawa theory for algebraic tori was developed by P. Schneider, and both our result and proof were suggested by his papers ([6], [7]).

Finally I would like to express my sincere gratitude to Prof. Takashi Ono for his warmhearted encouragement and dedicate this paper to him.

§1. Iwasawa L-function for an algebraic torus. In this section, we shall prepare the notations and assumptions which will be necessary below and define an Iwasawa L-function associated to the character module of an algebraic torus.

p: an odd prime number. μ_{p^n}: the group of p^n-th roots of unity. $\mu_{p^\infty} := \bigcup_{n\geq 1}\mu_{p^n}$. F: a totally real finite algebraic number field. T: an algebraic torus defined over F. $\hat{T}$: the group of rational characters of T. K: the minimal Galois splitting field of T. In the following, we assume the next condition (a).

(a) K is also totally real and p is unramified in $K/\boldsymbol{Q}$.

S: the set of primes of F which lie over p, ∞, or ramify in K/F. F_S: the maximal extension over F unramified outside S. For simplicity, we put $\widehat{T(p)} := \hat{T}\otimes_{\boldsymbol{Z}}\boldsymbol{Q}_p/\boldsymbol{Z}_p$ and denote by H_S and G the Galois groups $G(F_S/F(\mu_{p^\infty}))$ and $G(F(\mu_{p^\infty})/F)$ respectively. We shall use standard notations in Galois cohomology theory (e.g. [8]). Now, the Galois group G acts continuously on the Galois cohomology groups $H^i(H_S, \widehat{T(p)})$ ($i\geq 0$). For the Galois group G, we have the canonical decomposition $G=\Delta\times\Gamma$, $\Delta := G(F(\mu_p)/F)$, $\Gamma := G(F(\mu_{p^\infty})/F(\mu_p))\cong\boldsymbol{Z}_p$. If we denote by $H_i(T)$ the Δ-invariants of the Pontrjagin duals of $H^i(H_S, \widehat{T(p)})$, $H_i(T)$ are compact modules over completed group ring $\boldsymbol{Z}_p[[\Gamma]]$. Concerning the structures of $\boldsymbol{Z}_p[[\Gamma]]$-modules $H_i(T)$, we can prove the following.

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*) This paper is a part of my master's thesis in Tokyo Metropolitan University at 1988.