58. *Uniqueness and Existence of Viscosity Solutions of Generalized Mean Curvature Flow Equations*

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1. **Introduction.** We construct unique global continuous viscosity solutions of the initial value problem in $\boldsymbol{R}^n$ for a class of degenerate parabolic equations that we shall call *geometric*. A typical example is

$$(1) \qquad u_t-|\nabla u| \operatorname{div}\left(\frac{\nabla u}{|\nabla u|}\right)-\nu|\nabla u|=0 \qquad \left(u_t=\frac{\partial u}{\partial t},\ \nabla u=\operatorname{grad} u,\ \nu \in \boldsymbol{R}\right).$$

Our method is based on the comparison principle of viscosity solutions developed recently by Jensen [8] and Ishii [6]. However, as is observed from (1), our equation is singular at $\nabla u=0$ so we are forced to extend their theory to our situation.

The equation (1) has a geometric significance because γ-level surface $\Gamma(t)$ of u moves by its mean curvature when $\nu=0$ provided that ∇u does not vanish on $\Gamma(t)$. Such a motion of surfaces has been studied by many authors [1–5]. However, so far whole *unique* evolution families of surfaces were only constructed under geometric restrictions on initial surfaces such as convexity [3, 5] except $n=2$ [1, 4]. When $n=2$, Grayson [4] has shown that any embedded curve moved by its curvature never becomes singular unless it shrinks to a point. However when $n \geq 3$ even embedded surfaces may become singular before it shrinks to a point.

Our goal is to construct whole evolution family of surfaces even after the time when there appear singularities. This program is carried out by Angenent [1] when $n=2$. Contrary to [1] we avoid parametrization and rather understand surfaces as level sets of viscosity solutions of (1). Let $D(t)$ denote the open set of $x \in \boldsymbol{R}^n$ such that $u(x, t)>\gamma$. When the equation is geometric, it turns out that the family $(\Gamma(t), D(t))$ $(t \geq 0)$ is uniquely determined by $(\Gamma(0), D(0))$ and is independent of u and γ. By unique existence of viscosity solution of (1) we have a unique family of $(\Gamma(t), D(t))$ for all $t \geq 0$ provided $D(0)$ is bounded open and that $\Gamma(0)$ $(\subset \boldsymbol{R}^n \backslash D(0))$ is compact. As is expected, we conclude that $(\Gamma(t), D(t))$ becomes empty in a finite time provided $\nu \leq 0$. This extends a result of Huisken [5] where he proved that $\Gamma(t)$ disappears in a finite time provided $\Gamma(0)$ is a uniformly convex C^2 hypersurface.

In this note we state our main results almost without proofs; the details will be published elsewhere.

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