46. *A Sufficient Condition for Univalence and Starlikeness*

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Let A denote the class of functions of the form

$$f(z)=z+\sum_{n=2}^{\infty} a_n z^n$$

which are analytic in the unit disk $U=\{z : |z|<1\}$. For a function $f(z)$ belonging to the class A, Singh and Singh [3, Theorem 6] have proved the following result.

Theorem A. *If* $f(z)\in A$ *satisfies the condition*

$$1+\operatorname{Re}\frac{zf''(z)}{f'(z)}<\frac{3}{2} \qquad \text{in } U, \tag{1}$$

then

$$\operatorname{Re}\frac{zf'(z)}{f(z)}>0 \qquad \text{in } U.$$

Saitoh, Nunokawa, Fukui and Owa [2, Theorem 2] have improved Theorem A and have proved more precise result than Theorem A as the following:

Theorem B. *If* $f(z)\in A$ *satisfies the condition* (1), *then*

$$0<\operatorname{Re}\frac{zf'(z)}{f(z)}<\frac{1+\sqrt{3}}{2} \qquad \text{in } U. \tag{2}$$

In the present paper, the author improve the upper bound for $\operatorname{Re}(zf'(z)/f(z))$ in Theorem B.

Main theorem. *If* $f(z)$ A *satisfies the condition* (1), *then*

$$0<\operatorname{Re}\frac{zf'(z)}{f(z)}<\frac{4}{3} \qquad \text{in } U.$$

The inequalities are sharp.

Proof. Let us put

$$\frac{zf'(z)}{f(z)}=\frac{2(1-w(z))}{2-w(z)} \qquad z\in U. \tag{3}$$

Evidently $w(0)=0$.

Applying the same method as in the proof of [3, Theorem 6] and [1, p. 471], we have $|w(z)|<|z|<1$.

From (3), we have

$$|w(z)|=\left|\frac{2\left(1-\dfrac{zf'(z)}{f(z)}\right)}{2-\dfrac{zf'(z)}{f(z)}}\right|<|z|<1 \qquad \text{in } U.$$

This shows that