36. Some Results on Asymptotic Stability by Extending Matrosov's Theorems

By Teruyo WADA

Department of System Functions and Construction,
The Graduate School of Science and Technology, Kobe University

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1. Introduction. In this paper we present asymptotic stability theorems for ordinary differential equations by extending Matrosov's theorems [4].

Let us consider the following ordinary differential equation:

$$(1) \qquad \dot{x}=X(t,x), \qquad (X(t,0)\equiv 0),$$

where $X: \Gamma \to \boldsymbol{R}^n$ is a continuous function, $\Gamma=\boldsymbol{R}^+\times D$, $\boldsymbol{R}^+=[0,+\infty)$, and D is a domain in $\boldsymbol{R}^n$ satisfying $0\in D$.

Generalization of Liapunov's asymptotic stability theorem is considered by Barbashin and Krasovskiĭ (see [2] and [5]), Matrosov [4], LaSalle [3], Hatvani [1], Wada and Yamamoto [7] and etc. These results include the condition that the total derivative of a Liapunov function computed along the solutions of (1) is only negative semi-definite.

In the present paper, by extending Theorems 1.2 and 1.4 in [4], we establish theorems for (globally) asymptotic stability, (globally) equi-asymptotic stability and (globally) uniformly asymptotic stability as well as uniform stability of the zero solution of (1). In Theorems 1.2 and 1.4 of [4], Matrosov assumed that the function X, its partial derivatives $\partial X/\partial t$, $\partial X/\partial x_i$ $(i=1,2,\cdots,n)$, and the first and second partial derivatives of a Liapunov function V, that is, $\partial V/\partial t$, $\partial V/\partial x_i$, $\partial^2 V/\partial t^2$, $\partial^2 V/\partial t\partial x_i$, $\partial^2 V/\partial x_i\partial x_j$ $(i,j=1,2,\cdots,n)$, are continuous and bounded. In the foregoing paper [6], we extended Theorem 1.2 in [4] and gave uniform asymptotic stability theorems in which we generalized the above mentioned assumptions by Matrosov. Our resulting theorems in the present paper includes more useful conditions than the preceding paper's.

2. Theorems. For $\varepsilon>0$, B_ε is defined by $B_\varepsilon=\{x\in\boldsymbol{R}^n : \|x\|<\varepsilon\}$ and for $\alpha_1>\alpha_2>0$, the set $\Lambda(\alpha_1,\alpha_2)$ is defined by $\Lambda(\alpha_1,\alpha_2)=\{x\in\boldsymbol{R}^n : \alpha_1\le\|x\|\le\alpha_2\}$, where $\|x\|$ denotes the Euclidean norm of $x\in\boldsymbol{R}^n$. Let $C[A,E]$ be the family of all continuous functions from a set A into a set E. A function $a(\cdot)$ is called a function of class $\mathcal{K}$, i.e., $a\in\mathcal{K}$, if $a\in C[\boldsymbol{R}^+,\boldsymbol{R}^+]$ is a strictly increasing function with $a(0)=0$. The positive part $[x]_+$ of $x\in\boldsymbol{R}$ is defined by $[x]_+=\max\{0,x\}$, and the negative part $[x]_-$ of x is defined by $[x]_-=\max\{0,-x\}$. For a function $V\in C[\Gamma,\boldsymbol{R}]$ which is locally Lipschitzian in x, the total derivative $\dot{V}_{(1)}(t,x)$ of V with respect to (1) is defined by