27. *On the Existence of the Poles of the Scattering Matrix for Several Convex Bodies*

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1. Introduction. Let $\mathcal{O}$ be an open bounded set in $\boldsymbol{R}^3$ with smooth boundary Γ. We set

$$\Omega=\boldsymbol{R}^3-\bar{\mathcal{O}},$$

and suppose that Ω is connected. Consider the following acoustic problem

$$(1.1)\qquad \begin{cases} \square u(x,t)=\dfrac{\partial^2 u}{\partial t^2}-\Delta u=0 & \text{in } \Omega\times(-\infty,\infty) \\ Bu(x,t)=0 & \text{on } \Gamma\times(-\infty,\infty) \\ u(x,0)=f_1(x) & \\ \dfrac{\partial u}{\partial t}(x,0)=f_2(x) & \end{cases}$$

where $\Delta=\sum_{j=1}^3 \partial^2/\partial x_j^2$. As boundary operator B we shall consider the following two operators,

$$B_D=1 \quad \text{(Dirichlet condition)}$$

and

$$B_N=\sum_{j=1}^3 n_j(x)\partial/\partial x_j \quad \text{(Neumann condition)}$$

where $n(x)=(n_1(x), n_2(x), n_3(x))$ denotes the unit outer normal of Γ at x.

Denote by $\mathcal{S}_\dagger(z)$, $\dagger=D, N$, the scattering matrix for the scatterer $\mathcal{O}$ under the boundary condition $B_\dagger u=0$ (for the definition, see [6]). It is well known that $\mathcal{S}_\dagger(z)$ is an $\mathcal{L}(L^2(S^2))$-valued meromorphic function in the whole complex domain $\boldsymbol{C}$.

As to the modified Lax and Phillips conjecture,[1] that is, *when $\mathcal{O}$ is trapping, there exists $\alpha>0$ such that a slub domain $\{z\,;\operatorname{Im} z<\alpha\}$ contains an infinite number of poles of the scattering matrix*, we have a few examples. Especially for the Dirichlet boundary condition an obstacle consisting of two disjoint convex bodies is the only example ([2, 3]). The purpose of this note is to study the modified Lax and Phillips conjecture in the case that $\mathcal{O}$ consists of several disjoint strictly convex bodies. Our theorem gives a sufficient condition for the existence of such α, which is stated by means of an analytic function defined by using purely geometric informations of Ω.

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1) The original one is given in [6, page 158], but $\mathcal{O}$ considered in [4] is a counter example.