21. *Local Cohomology and the Absence of Poincaré Lemma in Tangential Cauchy-Riemann Complexes*

By Shinichi TAJIMA
Faculty of General Education, Niigata University

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We study the cohomology groups of the tangential Cauchy-Riemann complex with coefficients in microfunctions. In the section 1 of this note we give a sufficient condition for the non-vanishing of local cohomology groups with supports in certain closed subset. In the section 2 we show that, under some geometric condition, the Poincaré lemma fails for tangential Cauchy-Riemann complex with coefficients in microfunctions.

1. Local cohomology. Let X be a complex manifold of dimension n. Let $\mathcal{O}_X$ be the sheaf on X of holomorphic functions. Let Ω be an open subset of X and F the closed subset $X-\Omega$. We denote by $\mathcal{H}^k_F(\mathcal{O}_X)$, for $k=1,2,\cdots,n$, the local cohomology sheaves on X with supports in F. Let P be a boundary point of Ω.

Theorem 1. *Assume that there exists a germ of complex subvariety V of codimension q passing through the point P which satisfies the following conditions*

(i) *V is a complete intersection in U*

(ii) $(V\cap U)\cap\Omega=\phi$

for some neighborhood U of P.

Then at least one of the local cohomology groups $\mathcal{H}^1_F(\mathcal{O}_X)$, $\mathcal{H}^2_F(\mathcal{O}_X)$, $\cdots$, $\mathcal{H}^q_F(\mathcal{O}_X)$ does not vanish at P.

The proof is based on the fundamental properties of the generalized Bochner-Martinelli form (cf. [10]).

The following corollary is a natural generalization of a result of Andreotti-Norguet [1].

Corollary 2. *If, under the assumptions of Theorem 1, $\mathcal{H}^1_F(\mathcal{O}_X)_P=\mathcal{H}^2_F(\mathcal{O}_X)_P=\cdots=\mathcal{H}^{q-1}_F(\mathcal{O}_X)_P=0$, then $\mathcal{H}^q_F(\mathcal{O}_X)_P\neq 0$.*

Regarding $\{P\}$ as a complex submanifold of codimension n, we have the following

Corollary 3. *Let P be a boundary point of Ω. If $\mathcal{H}^2_F(\mathcal{O}_X)_P=\cdots=\mathcal{H}^n_F(\mathcal{O}_X)_P=0$, then $\mathcal{H}^1_F(\mathcal{O}_X)_P\neq 0$.*

Note that Corollary 3 is a local cohomological version of a result of Hörmander Theorem (Th. 4.2.9 of [5]).

2. Tangential Cauchy-Riemann complex. Let $\Omega=\{z\,|\,\rho(z,\bar{z})<0\}$ be a domain in X with real analytic boundary N. Here ρ is a real-valued real analytic function. (We assume that the gradient grad ρ of ρ does not vanish on N.) F denotes the closed subset $\{z\,|\,\rho(z,\bar{z})\geqq 0\}$. Let Y be a com-