104. The Cauchy Problem for a Class of Hyperbolic Operators with Triple Characteristics

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1. Introduction. In the C^∞ category the well-posedness of the Cauchy problem for hyperbolic operators depends in general on the behaviour of the lower order terms. When the characteristic roots are at most double, necessary and (almost) sufficient conditions have been given by Ivrii-Petkov [5], Ivrii [4] and Hörmander [2]. For higher order multiplicities results on the (microlocal) Cauchy problem have been proved by Bernardi [1] in the involutive case and by Nishitani [6] in the "effectively" hyperbolic case. In comparison with these last two cases the Levi conditions in a non-involutive and "non-effectively" hyperbolic situation seem to be much more involved and that is the reason why we restricted ourselves to multiplicity of order three.

Let us now introduce our notations. Let $\Omega\subset \boldsymbol{R}^{n+1}$ an open subset, $x=(x_0, x_1, \cdots, x_n)\in\Omega$, $D_{x_j}=(1/i)\partial_{x_j}$, $j=0, \cdots, n$, $x=(x_0, x')$. Let $P(x, D_x)=P_m(x, D_x)+P_{m-1}(x, D_x)+\cdots$, be a hyperbolic differential operator of order m. We denote by Γ_ρ, $\rho\in\Omega\times \boldsymbol{R}^n\setminus\{0\}$, the hyperbolicity cone of P in ρ and by Γ_ρ^σ the polar of Γ with respect to the symplectic two-form $\sigma=d\xi\wedge dx=d\omega$, ω the canonical one-form. See [7] for the definition of Γ_ρ. We recall the definition of the subprincipal symbol of $P: P_{m-1}^s(x, \xi)=P_{m-1}(x, \xi)+(1/2)\sum_{j=0}^n \partial^2_{x_j\xi_j} p_m(x, \xi)$. It is invariantly defined at double characteristic points of p_m. If q is a hyperbolic operator with double characteristics we note by F_q the fundamental matrix of q_2, the principal symbol of q, and by $\mathrm{Tr}^+ F_q=\sum \lambda_j$, where $\pm i\lambda_j\in sp(F_q)$. See [3] for precise definitions. We now state our results:

2. Results. We shall make the following assumptions on P.

H1) The principal symbol of P, $p_m(x, \xi)$ is hyperbolic with respect to ξ_0.

H2) The characteristic roots of $\xi_0\to P_m(x, \xi_0, \xi')$ have multiplicities at most of order 3 and the triple characteristic set $\Sigma=\{(x, \xi)\in\Omega\times \boldsymbol{R}^n\setminus\{0\}\,|\, p_m(x, \xi)=dp_m(x, \xi)=d^2p_m(x, \xi)=0\}$ is a C^∞ manifold such that rank $\sigma|_\Sigma=$ const and ω does not vanish identically on $T\Sigma$.

Let $\rho\in\Sigma$:

H3)$_\rho$ Denote by $\mathrm{T}_\rho(\Omega\times \boldsymbol{R}^n\setminus\{0\})\ni\delta z\to P_{m,\rho}(\delta z)$ the localization of P_m at ρ (see e.g. [7]). Then

(i) $P_{m,\rho}(\delta z)=L_1(\delta z)Q_2(\delta z)$ where $L_1(\delta z)=\delta\xi_0-l_1(\delta x, \delta\xi')$, l_1 being a real linear form in $(\delta x, \delta\xi')$.

(ii) $Q_2(\delta z)$ is a real hyperbolic quadratic form such that: