100. A Cohomological Construction of Swan Representation over the Witt Ring. II

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This is continued from [0].

3. In this section we give the construction of Swan representations. Let $K=k((T^{-1}))$ be a complete discrete valuation field and M be a finite Galois extension of K with Galois group G. N. M. Katz proved the following

Theorem ([7] Theorem (1.4.1)). *There exists a canonical finite etale Galois covering*

$$U \longrightarrow \boldsymbol{G}_{m,k}=\operatorname{Spec} k[T, T^{-1}]$$

which satisfies the following properties.

(1) $$U \otimes_{G_{m,k}} \operatorname{Spec} k((T^{-1})) \simeq \operatorname{Spec} M$$

(2) $U \otimes_{G_{m,k}} \operatorname{Spec} k((T))$ *is a disjoint union of the spectra of tamely ramified extensions of* $k((T))$.

We denote by $X \xrightarrow{g} \boldsymbol{P}_k^1$ the compactification of $U \longrightarrow \boldsymbol{G}_{m,k}$. Note that g factors as $X \longrightarrow \boldsymbol{P}_m^1 \longrightarrow \boldsymbol{P}_k^1$, where m denotes the residue field of M. We denote by D_0 (resp. D_∞) the inverse image of $T=0$ (resp. $T=\infty$) with reduced scheme structure. Then $X \backslash U = D_0 \coprod D_\infty$. Let $W\Omega_X^{\cdot}(\log D_0 - \log D_\infty)$ be the de Rham-Witt complex with logarithmic poles along D_0 and with minus logarithmic poles along D_∞ as in §1. As D_0 and D_∞ are stable under the action of G, $\sigma \in G = \operatorname{Gal}(U/\boldsymbol{G}_{m,k})$ acts on the free W-module

$$H^1(X, W\Omega_X^{\cdot}(\log D_0 - \log D_\infty))$$

by transportation of structures. The following Proposition shows that this is the desired space of the Swan representation of G.

Proposition. *The trace of the action of* $\sigma \in G$ *on*

$$H^1(X, W\Omega_X^{\cdot}(\log D_0 - \log D_\infty))$$

coincides with $Sw_G(\sigma)$.

In the following we denote the alternating sum of the trace of the action of σ on free $W(k)$-modules by

$$\operatorname{Tr}(\sigma : R\Gamma(X, \quad)) := \sum_{q \geq 0} (-1)^q \operatorname{Tr}(\sigma : H^q(X, \quad)).$$

By Lemma and exact sequences (∗∗) in §1, it suffices to show

$$\operatorname{Tr}(\sigma : R\Gamma(X, W\Omega_X^{\cdot}) = \begin{cases} d^\sigma + (-Sw_G(\sigma) + f) & \text{for } \sigma \in I, \\ 0 & \text{for } \sigma \notin I, \end{cases}$$

where d^σ denotes the degree of the closed subscheme of D_0 fixed by σ and $f=[m:k]$ coincides with degree of D_∞ over k.

The proof of this formula is the same as the proof of the Weil formula [4] §5: The case $\sigma=1$ is the Hurwitz formula. The case $\sigma \neq 1$ is deduced