97. Selberg Trace Formula for Odd Weight. I

By Shigeki AKIYAMA
Department of Mathematics, Kobe University

(Communicated by Shokichi IYANAGA, M. J. A., Nov. 14, 1988)

§0. **Introduction.** Let Γ be a fuchsian group of the first kind which does not contain the element $-I=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$. The first aim of this note is to rewrite the Selberg trace formula for odd weight and the group Γ, in a form which makes clear the difference between the contributions of regular cusps and of irregular cusps. Such a difference is already known in the case of the dimension formula for weight ≥ 3 and we are interested to know how it appears in general case. Our second aim is to improve the dimension formula for weight one. At the symposium "Automorphic Forms and Related Topics" at Research Institute of Mathematical Science, Kyoto University on 1987, Tanigawa, Hiramatsu and the author made a report on the trace formula for Hecke operators in the case of weight one. In the special case of the report, we gave a dimension formula of the space of cusp forms of weight one, using the residue of the Selberg type zeta function ([5]). But the formula is unsatisfactory because the "zeta" has no functional equation. In this note, we give a dimension formula of weight one, using more natural "zeta" function which has a functional equation.

§1. **Notation.** Let $\boldsymbol{H}$ be the complex upper half plane and $T=\boldsymbol{R}/(2\pi)$. We put $\tilde{\boldsymbol{H}}=\boldsymbol{H}\times T$, $G=SL(2,\boldsymbol{R})$ and $\tilde{G}=G\times T$. Then $\tilde{G}$ acts transitively on $\tilde{\boldsymbol{H}}$ in the following way:

$$(g,\alpha)\cdot(z,\phi)=(g\cdot z,\phi+\arg j(g,z)-\alpha)\qquad (g,\alpha)\in\tilde{G},\quad (z,\phi)\in\tilde{\boldsymbol{H}},$$

where $g=\begin{bmatrix}a & b\\ c & d\end{bmatrix}\in G$, $g\cdot z=\dfrac{az+b}{cz+d}\in\boldsymbol{H}$ and $j(g,z)=cz+d$. With the involution $\xi(z,\phi)=(-\bar{z},-\phi)$, the triple $(G,\boldsymbol{H},\xi)$ is the weakly symmetric Riemannian space. The ring of $\tilde{G}$ invariant differential operators on this space is generated by Δ and $\partial/\partial\phi$ where

$$\Delta=y^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+y\frac{\partial}{\partial x}\frac{\partial}{\partial\phi}.$$

Let Γ be the discrete subgroup of G not containing $-I$. We identify G with $G\times\{0\}$, and Γ with $\Gamma\times\{0\}$. Take a unitary representation χ of Γ of degree ν ($<\infty$). Let $\kappa_1,\kappa_2,\cdots,\kappa_\omega$ be the complete representatives of Γ-inequivalent cusps of $\Gamma\backslash\boldsymbol{H}$. Γ_i denotes the stabilizer of κ_i, and $\Gamma_i^0=\Gamma_i\cap\ker\chi$. We will consider the cusps form of $\Gamma\backslash\boldsymbol{H}$, so take χ under the condition $[\Gamma_i\Gamma_i^0]<\infty$ for $i=1,2,\cdots,\omega$. Take $\sigma_i\in G$ such that $\sigma_i\infty=\kappa_i$, satisfying the following condition:

If κ_i is regular then $\Gamma_\infty=\sigma_i^{-1}\Gamma_i\sigma_i$ is generated by $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$;