89. *Decreasing Streamlines of Solutions and Spectral Properties of Linearized Operators for Semilinear Elliptic Equations*

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(Communicated by Kôsaku YOSIDA, M. J. A., Nov. 14, 1988)

§1. Introduction. Let $\Omega\subset\boldsymbol{R}^N$ be a bounded domain with a smooth boundary $\partial\Omega$ and $f:\boldsymbol{R}\to\boldsymbol{R}$ be a C^1 function. We consider the semilinear elliptic equation

(1) $$-\Delta u=f(u),\quad u>0 \text{ (in } \Omega),\quad u=0 \text{ (on } \partial\Omega).$$

Then the linearized operator around the solution $u=u(x)\in C^2(\Omega)\cap C^0(\bar{\Omega})$ is given by $A\equiv A(u)=-\Delta-f'(u)$ in $L^2(\Omega)$ with $D(A)=H_0^1\cap H^2(\Omega)$. In the previous work [1], we have given some streamlines in Ω along which the solution decreases, when Ω is a symmetric domain in $\boldsymbol{R}^2$. There, we restricted ourselves to the mild solution, that is, the case when the second eigenvalue $\mu_2=\mu_2(u)$ of $A=A(u)$ is positive. In this article, we shall note that conversely, the decreasing streamlines of the solution contain some information about the eigenvalues of $A(u)$.

Thus, we suppose that the domain is the unit ball: $\Omega=\{|x|<1\}\subset\boldsymbol{R}^N$. Then from [5], every solution $u=u(x)$ of (1) is radial: $u=u(|x|)$ and $u_r<0$ for $0<r=|x|<1$. Therefore, the set of eigenvalues $\sigma(u)$ of $A(u)$ is divided as $\sigma(u)=\bigcup_{m=0}^{\infty}\sigma_m(u)$ according to the principle of separation of variables. Namely, let $\{\rho_m\}_{m=0}^{\infty}$ $(0=\rho_0<\rho_1<\rho_2<\cdots)$ be the eigenvalues of $-\Lambda$, where Λ denotes the Laplace-Beltrami operator on $S^{N-1}=\{|x|=1\}$. In fact we have $\rho_m=m(2\nu+m)$, where $2\nu=N-2$. Further, multiplicity κ_m of ρ_m (and hence that of $\mu\in\sigma_m(u)$) is as follows: for $N=2$ we have $\kappa_m=1$ $(m=0)$ and $\kappa_m=2$ $(m\geqq 1)$; for $N>2$ we have $\kappa_m=(2m+N-2)(m+N-3)!/(N-2)!\,m!$ (see, e.g. [9]). Then $\sigma_m(u)$ denotes the set of eigenvalues of the ordinary differential operator $A_m(u)=-(d^2/dr^2)-((N-1)/r)(d/dr)-c(r)+(\rho_m/r^2)$ with $(d/dr)\cdot|_{r=0}=\cdot|_{r=1}=0$, where $c(r)=f'(u)$.

Now, for these sets $\sigma_m(u)$ $(m=0,1,2,\cdots)$, we claim the following, where $\boldsymbol{R}_+=(0,\infty)$:

Theorem. *If $f(\boldsymbol{R}_+)\subset\boldsymbol{R}_+$, then $\sigma_m(u)\cap(-\infty,0]=\phi$ for $m\geqq 1$. In particular,* dim Ker $A(u)$ *is at most* 1 *for any solution u on the ball $\Omega=\{|x|<1\}\subset\boldsymbol{R}^N$, provided that $f(\boldsymbol{R}_+)\subset\boldsymbol{R}_+$.*

§2. Proof of Theorem. Set $\sigma_m(u)=\{\mu_j^m\,|\,j=0,1,2,\cdots\}$ with $-\infty<\mu_0^m<\mu_1^m<\cdots$. Since $\rho_{m'}>\rho_m$ if $m'>m$, we have $\mu_0^1<\mu_0^2<\cdots$ and hence we have only to prove that $\mu_0^1>0$.

The eigenfunction φ of $A(u)$ corresponding to μ_0^1 is of the form $\varphi(x)=$