87. The Period Map of a 4-parameter Family of K3 Surfaces and the Aomoto-Gel'fand Hypergeometric Function of Type (3, 6)[†]

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We show that one of Aomoto-Gel'fand hypergeometric functions ([4]) can be interpreted as the period map of a 4-dimensional family of $K3$ surfaces, of which the target is the 4-dimensional Hermitian symmetric bounded domain of type IV. The corresponding system of differential equations has six linearly independent solutions which are quadratically related. (Such systems are recently studied in [10].) This fact confers an algebro-geometric decoration to the Aomoto-Gel'fand functions as the relation between the elliptic modular function and the corresponding equation does to the Gauss hypergeometric function. Details will be given in [6].

We describe a family of $K3$ surfaces. Let

$$l_j=\{(t^1,t^2,t^3)\in \boldsymbol{CP}^2 \mid v_{1j}t^1+v_{2j}t^2+v_{3j}t^3=0\} \qquad (0\leqq j\leqq 6)$$

be six lines in general position in the complex projective plane $\boldsymbol{CP}^2$ with homogeneous coordinates (t^1,t^2,t^3) and let $S(l)$ be the minimal smooth model of the two-fold cover $S'(l)$ of $\boldsymbol{CP}^2$ branching along the line configuration $l=\{l_1,\cdots,l_6\}$. For a fixed l, the surface $S(l)$ is a $K3$ surface, i.e., there is a unique holomorphic 2-form

$$\eta(l)=\prod_{j=1}^{6}(v_{1j}s^1+v_{2j}s^2+v_{3j})^{-1/2}ds^1\wedge ds^2, \tag{1}$$

up to constant multiplication, and the rank of the second homology group $H_2(S(l),\boldsymbol{Z})$ is 22. In this case, there are 16 linearly independent cycles; 15 exceptional curves coming from the 15 double points of $S'(l)$ and a section when considered $S(l)$ as an elliptic surface over $\boldsymbol{CP}^1$. We can take a system $\gamma_1'(l),\cdots,\gamma_6'(l)\in H_2(S(l)\boldsymbol{Z})$ of six (transcendental) cycles orthogonal to the algebraic cycles such that there exists another system $\gamma_1(l),\cdots,\gamma_6(l)\in H_2(S(l),\boldsymbol{Z})$ which is dual to γ_j' $(1\leqq j\leqq 6)$, i.e., $\gamma_i'\cdot\gamma_j=\delta_{ij}$ (Kronocker's symbol) and that its intersection matrix $(\gamma_i'\cdot\gamma_j')$ $(1\leqq i,j\leqq 6)$ takes the fixed form $I=(I_{ij})$, which is symmetric, integral and with the signature $(2+,4-)$. The vector $\omega(l)=(\omega_1(l),\cdots,\omega_6(l))$, where $\omega_j(l)=\int_{\gamma_j(l)}\eta(l)$ $(1\leqq j\leqq 6)$, is called the period of $S(l)$ and it satisfies the Riemann relation and the Riemann inequality as follows

$$\sum_{i,j}I_{ij}\omega_i(l)\omega_j(l)=0 \tag{2}$$

$$\sum_{i,j}I_{ij}\omega_i(l)\bar{\omega}_j(l)>0. \tag{3}$$

[†] Dedicated to Professor Hisasi MORIKAWA on his 60th birthday.

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