85. A Cohomological Construction of Swan Representations over the Witt Ring. I

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0. Let K be a complete discrete valuation field with residue field k. We assume k is a perfect field of characteristic $p>0$. For a finite Galois extension M/K with Galois group G, the Swan character $Sw_G: G\to \boldsymbol{Z}$ is defined as follows.

$$Sw_G(\sigma)=\begin{cases}(1-v_M(\sigma(\pi_M)-\pi_M))\cdot f & \text{for } 1\neq\sigma\in I,\\ 0 & \text{for } \sigma\notin I,\end{cases}$$

$$Sw_G(1)=-\sum_{1\neq\sigma\in G} Sw_G(\sigma).$$

Here I denotes the inertia group, π_M a prime element of M, v_M the normalized valuation of M and f the degree of the residue field extension. Then it is a classical result that Sw_G is a character of a linear representation of G and that it can be defined over the l-adic field $\boldsymbol{Q}_l$ $(l\neq p)$ (resp. the fraction field of the Witt ring $W(k)$) [2], [8]. We call it the Swan representation of G and denote it by $Sw_{G,l}$ (resp. $Sw_{G,p}$).

In this note we construct $Sw_{G,p}$ cohomologically (or geometrically) *when K is of equal characteristic p*. The construction of $Sw_{G,l}$ $(l\neq p)$ was done by Katz [7]. He uses his theory of canonical extension (cf. Theorem in §3) and the machinery of l-adic etale cohomology. Instead of l-adic etale cohomology, we use a new theory of de Rham-Witt complex with logarithmic poles, which supplies us nice p-adic cohomology for open varieties. Recently, general theory of crystals with logarithmic poles has been developed independently by G. Faltings [1] and K. Kato [6].

The content of this note is as follows. In §1–2 we introduce the de Rham-Witt complex with logarithmic poles, and construct $Sw_{G,p}$ in §3. The author would like to thank Prof. K. Kato, whose observation explained in §2 is the key to the definition of de Rham-Witt complex with logarithmic poles.

1. In this and next section we introduce the de Rham-Witt complex with logarithmic poles as a preparation for §3. Here we give a short exposition concerning what is necessary in §3, and full details will be treated elsewhere. In this note we always consider sheaves and cohomologies in the etale topology.

Let k be a perfect field of characteristic $p>0$, X a smooth scheme over k and D a reduced normally crossing divisor in X. We will define sheaves of complexes $W_n\Omega^{\cdot}_X(\log D)$ (resp. $W_n\Omega^{\cdot}_X(-\log D)$), which we shall call the de Rham-Witt complex with logarithmic poles (resp. with minus logarithmic