82. *On Generalization of G_δ Diagonal and Metrization*

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(Communicated by Shokichi IYANAGA, M. J. A., Oct. 12, 1988)

Introduction. J. Chaber's result, [6], that countably compact spaces with G_δ diagonal are compact, has inspired us to define the conditions (α_i), $i=1,2$ as follows. (See J.G. Ceder's characterization of G_δ diagonal, [5].)

(α_1) (resp. (α_2)): There is a sequence $\{\mathcal{U}_n\}_n$ of semi-open covers with $\bigcap_n St(x,\mathcal{U}_n)=\{x\}$, (resp. $\bigcap_n \overline{St(x,\mathcal{U}_n)}=\{x\}$) for each x in the topological space under consideration, where $St(x,\mathcal{U}_n)=\bigcup\{U\in\mathcal{U}_n \mid x\in U\}$. We call such a sequence $\{\mathcal{U}_n\}_n$ an (α_1) (resp. (α_2)) sequence.

According to [1], a cover $\mathcal{U}$ of a space x is called *semi-open*, if $x\in \operatorname{Int} St(x,\mathcal{U})$ for $x\in X$. Clearly (α_2) and G_δ diagonal property [5] imply (α_1).

1. Implication of the conditions (α_i), $i=1,2$. Refer [2], [11] and [12] respectively for the notions of point countable type, q and wM.

Lemma 1.1. *A regular q space X with (α_1) is first countable.*

Proof. Let $x\in X$, $\{G_n\}_n$ be a q sequence at x and $\{\mathcal{U}_n\}_n$ be an (α_1) sequence in X. By induction, we can obtain a sequence $\{H_n\}_n$ of open sets with $x\in H_{n+1}\subset \overline{H}_{n+1}\subset H_n\cap G_{n+1}\cap \operatorname{Int} St(x,\mathcal{U}_{n+1})$ for each n. Now, $\{H_n \mid \geqslant 1\}$ forms a local base at x. Q.E.D.

Theorem 1.2. *A regular q space with (α_1) is an open continuous image of a metrizable space.*

Proof. Apply the Lemma 1.1 and a result of Ponomarev and Hanai [13], [8]. Q.E.D.

Theorem 1.3. *If a regular q space with (α_1) is a quotient image of a locally compact, separable and metrizable space, then the space is locally compact, separable and metrizable.*

Proof. Apply the Lemma 1.1 and a result of A. H. Stone, [15].
Q.E.D.

Note 1.4. Since a T_1 space of point countable type is a q space [9], we can replace "q space" by "space of point countable type" in the above theorem.

Theorem 1.5. *A wM space X is metrizable, iff it has (α_2).*

Proof. The condition is clearly necessary. To prove that the condition is sufficient, let X have (α_2) and $\{\mathcal{U}_n\}_n$ be an (α_2) sequence. Let $\{\mathcal{G}_n\}_n$ be a wM sequence in X. Let $\mathcal{H}_n=\mathcal{U}_n\wedge\mathcal{G}_n$ for each $n\geqslant 1$, where $\mathcal{U}_n\wedge\mathcal{G}_n=\{U\cap G \mid U\in\mathcal{U}_n \text{ and } G\in\mathcal{G}_n\}$. Then $\{\mathcal{H}_n\}_n$ is also an (α_2) sequence. We may assume that $\mathcal{H}_{n+1}\prec\mathcal{H}_n$ (i.e. $\mathcal{H}_{n+1}$ refines $\mathcal{H}_n$) for each n. To claim that $\{\mathcal{H}_n\}_n$ is a semi-development of X, we show that $\{St(x,\mathcal{H}_n) \mid n\geqslant 1\}$ is a network at x