80. *Some Oscillation Criteria for Second Order Nonlinear Ordinary Differential Equations with Damping*

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1. Introduction. In this paper we consider the oscillatory behavior of the solutions of the second order nonlinear differential equation with damping

(1) $$(r(t)x')'+p(t)x'+q(t)f(x)=0, \qquad t\in[0,\infty),$$

where $r, p, q \in C[0,\infty)$, $r(t)>0$, and p, q are allowed to take on negative values for arbitrarily large t, $f\in C(\boldsymbol{R})$, $xf(x)>0$ for $x\neq 0$. We restrict our attention to solutions of (1) which exist on some interval $[\tau_0,\infty)$.

For the second order linear differential equation:

(*) $$x''+q(t)x=0,$$

the well-known theorem of Wintner [3] for the equation (*) to be oscillatory. Later more general theorems were established by considering weighted averages of the integral of q.

Recently, by the use of completing square and averaging technique, Yan [2] gave the following oscillation theorem for the equation:

(2) $$(r(t)x')'+p(t)x'+q(t)x=0, \qquad t\in[0,\infty).$$

Theorem. *If there exist $\alpha\in(1,\infty)$ and $\beta\in[0,1)$ such that*

(3) $$\overline{\lim_{t\to\infty}}\, t^{-\alpha}\int_{t_0}^{t}(t-\tau)^{\alpha}\tau^{\beta}q(\tau)d\tau=\infty,$$

(4) $$\overline{\lim_{t\to\infty}}\int_{t_0}^{t}[(t-\tau)p(\tau)\tau+\alpha\tau-\beta(t-\tau)]^2(t-\tau)^{\alpha-2}\tau^{\beta-2}dt<\infty,$$

then (1) *is oscillatory.*

Moreover Yan [1] established two theorems as criteria for the oscillation of (2) when (3) or (4) is not satisfied.

We extend his results for (2) in [1] to the equation (1).

2. Main results. We consider the equation (1) under the following assumption.

Assumption. (a) r, p, and q are continuous on $[0,\infty)$, and $r(t)>0$.

(b) $f:\boldsymbol{R}\to\boldsymbol{R}$ is continuously differentiable such that $xf(x)>0$ $(x\neq 0)$, and $f'(x)\geq k>0$ for some constant k. Our results are as follows:

Theorem 1. *Suppose that there exist a positive continuously differentiable function $h(t)$ on $[0,\infty)$ and a constant $\alpha\in(1,\infty)$ such that*

(5) $$\overline{\lim_{t\to\infty}}\, t^{-\alpha}\int_{t_0}^{t}H_k(t,\tau)d\tau=\infty,$$

where $H_k(t,\tau)=(t-\tau)^{\alpha}h(\tau)q(\tau)$