79. On the Isomonodromic Deformation of Certain Pfaffian Systems Associated to Appell's Systems (F_2), (F_3)

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§ 1. **Introduction.** The purpose of this paper is to derive systems of isomonodromic deformation equations associated to Appell's systems (F_2), (F_3), which we shall present in forms that are transformed to Pfaffian systems.

In 1880, P. Appell, generalizing Gauss' hypergeometric equation to the case of two variables, introduced four systems [1]:

$$(F_1)\ \begin{cases}\theta(\theta+\theta'+\gamma-1)z-x(\theta+\theta'+\alpha)(\theta+\beta)z=0\\ \theta'(\theta+\theta'+\gamma-1)z-y(\theta+\theta'+\alpha)(\theta'+\beta')z=0,\end{cases}$$

$$(F_2)\ \begin{cases}\theta(\theta+\gamma-1)z-x(\theta+\theta'+\alpha)(\theta+\beta)z=0\\ \theta'(\theta'+\gamma'-1)z-y(\theta+\theta'+\alpha)(\theta'+\beta')z=0,\end{cases}$$

$$(F_3)\ \begin{cases}\theta(\theta+\theta'+\gamma-1)z-x(\theta+\alpha)(\theta+\beta)z=0\\ \theta'(\theta+\theta'+\gamma-1)z-y(\theta'+\alpha')(\theta'+\beta')z=0,\end{cases}$$

$$(F_4)\ \begin{cases}\theta(\theta+\gamma-1)z-x(\theta+\theta'+\alpha)(\theta+\theta'+\beta)z=0\\ \theta'(\theta'+\gamma'-1)z-y(\theta+\theta'+\alpha)(\theta+\theta'+\beta)z=0,\end{cases}$$

where $z=z(x,y)$ is unknown function and $\theta=x\partial/\partial x$, $\theta'=y\partial/\partial y$. It is known that Appell's system (F_1) is transformed into a Pfaffian system on $P_2(C)$:

(1.1) $df=(\sum_{i=1}^{6}A_i(dF_i(x,y)/F_i(y,x)))f$, where $f={}^t(z, x\partial z/\partial x, y\partial z/\partial y)$, $A_i\in gl(3,C)$ and F_i's are the defining equations of singular locus of (F_1). (For example see [2].) B. Klares studied the isomonodromic deformation equation of the completely integrable Pfaffian system associated to this system (1.1) in [3]. It is also known that, as well as (F_1), Appell's systems (F_2), (F_3) are transformed into Pfaffian systems of type:

(1.2) $df=(\sum_{i=1}^{6}A_i(dF_i(x,y)/F_i(x,y)))f$, where $f={}^t(z, x\partial z/\partial x, y\partial z/\partial y, xy\partial^2 z/\partial x\partial y)$, $A_i\in gl(4,C)$ and F_i's are the defining equations of singular locus of the systems $(i=1,2,\cdots,6)$.

The author will present the systems of isomonodromic deformation equations associated to Pfaffian systems (1.2) by the same method as in [3]. Main results of this note are obtained in [4].

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§ 2. **Pfaffian systems satisfying Appell's systems (F_2), (F_3).** Let p: $(C^3)^*\to P_2(C)$ be a canonical projection and (X,Y,Z) be a homogeneous coordinates on $P_2(C)$ with $x=X/Z$, $y=Y/Z$.

Proposition 1. *Appell's system (F_2) is transformed into the following completely integrable Pfaffian system on $P_2(C)$:*