77. *On Pathwise Projective Invariance of Brownian Motion.* II

By Shigeo TAKENAKA
Department of Mathematics, Nagoya University

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In part I, we have obtained a measure preserving group action which is isomorphic to $SL(2, \boldsymbol{R})$ in the path space of Brownian motion $\{B(t;\omega)\}$;

$$B^g(t;\omega)=(ct+d)B\left(\frac{at+b}{ct+d};\omega\right)-ctB\left(\frac{a}{c};\omega\right)-dB\left(\frac{b}{d};\omega\right), \tag{1}$$
$$g=\begin{pmatrix} a, & b \\ c, & d \end{pmatrix}\in SL(2,\boldsymbol{R}).$$

From this action we have deduced a symmetric property called P. Lévy's *projective invariance of Brownian motion.*

In this part we, using the terms of theory of unitary representation, determine the class of the above action a *discrete series representation* of index 2.

§4. Stochastic integral of Wiener type. Let $\{X(t;\omega);t\in\boldsymbol{R}\}$ be a Gaussian process with continuous path and $\varphi\in\mathcal{D}(\boldsymbol{R})$ be a test function. Define an integral of Wiener type;

$$I(\varphi;\omega)\equiv-\int_{\boldsymbol{R}}\varphi'(t)X(t;\omega)dt. \tag{6}$$

An inner product is defined by

$$(\varphi,\psi)\equiv\boldsymbol{E}[I(\varphi;\omega)\overline{I(\psi;\omega)}]=\iint\varphi'(t)\overline{\psi'(t)}\boldsymbol{E}X(t)X(s)dtds. \tag{7}$$

Let us denote L_X^2 the completion of $\mathcal{D}(\boldsymbol{R})$ by above inner product. Then $I(\cdot)$ becomes an isometry form L_X^2 into $L^2(\Omega)$.

Example 1. In case of Brownian motion, L_X^2 is $L^2(\boldsymbol{R}, dx)$ and the isometry $I(\cdot)$ is nothing but the Wiener integral.

Example 2. Let us consider a self-similar process X^α of index α (see §3). The inner product is

$$(\varphi,\psi)=\frac{1}{2}\iint\varphi'(t)\overline{\psi'(s)}\{|t|^\alpha+|s|^\alpha-|t-s|^\alpha\}dtds$$
$$=\frac{1}{2}\alpha(\alpha-1)\iint\varphi(t)\overline{\psi(s)}\,|t-s|^{\alpha-2}\,dtds.$$

We obtain the above formula in the sense of generalized functions, the function $|t-s|^{\alpha-2}$ is accordingly considered a pseudo function (Gel'fand etc. [6]). The above inner product space is used as the space of supplementary series representation of $SL(2,\boldsymbol{R})$. Therefore this example suggests us a certain connection between self-similar process and supplementary series representation of $SL(2,\boldsymbol{R})$.