71. *Eigenvalues and Eigenvectors of Supermatrices*

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§1. **Introduction and preliminaries.** The theories of linear algebra and analysis over a Grassmann algebra have been developed and are a base of the theory of supermanifolds, Lie supergroups and Lie superalgebras, which are extensively used in modern physics. In his excellent book [1], Berezin treated diagonalization of supermatrices, but he proved it only in a direct way using induction on the number of generators of Grassmann algebras. In this note we study the eigenvalue problem of supermatrices in a general and natural manner by introducing the notions of (super) eigenvalue and eigenvector. We need to consider odd eigenvectors as well as even ones, and corresponding to them two kinds of eigenvalues appear. Starting with the ordinary eigenvalues of the body of a given supermatrix we can find its supereigenvalues by the perturbation method. Our method gives an efficient algorithm to compute eigenvalues and eigenvectors, and we demonstrate this by a simple example. The diagonalization of supermatrices will be done as a by-product of the solution of the eigenvalue problem.

Let Λ be a Grassmann algebra over the complex numbers $\boldsymbol{C}$, generated by a finite or infinite number of odd elements. The algebra Λ is a direct sum of the even part Λ_0 and the odd part Λ_1. The body of an element a of Λ is denoted by $\tilde{a}$. Then the $\tilde{}$ is a mapping of Λ to $\boldsymbol{C}$.

Let p and q be nonnegative integers and let $n=p+q$. By an even (resp. odd) vector we mean a column $(x_1, \cdots, x_p, x_{p+1}, \cdots, x_{p+q})^T$, where x_i is in Λ_0 (resp. Λ_1) for $i=1, \cdots, p$ and in Λ_1 (resp. Λ_0) for $i=p+1, \cdots, p+q$. We consider a supermatrix M of the form $M=\begin{bmatrix} A & B \\ C & D \end{bmatrix}$, where A (resp. D) is a $p\times p$-matrix (resp. $q\times q$-matrix) whose entries are in Λ_0 and B (resp. C) is a $p\times q$-matrix (resp. $q\times p$-matrix) whose entries are in Λ_1. If x is an even (resp. odd) vector, then Mx is an even (resp. odd) vector.

A supernumber $\lambda\in\Lambda_0$ is called an *eigenvalue* of a supermatrix M, if there exists a vector x such that $Mx=\lambda x$ and $\tilde{x}=(\tilde{x}_1, \cdots, \tilde{x}_{p+q})^T$ is nonzero. This vector x is called an *eigenvector* corresponding to λ. If x is even (resp. odd), we say λ is an eigenvalue of the first (resp. second) kind.

§2. **Eigenvalues of unmixed matrices.** In this section we consider the case where $p=0$ or $q=0$, and therefore the supermatrices are usual matrices over Λ_0. Let $f(X)=a_0+a_1X+a_2X^2+\cdots+a_nX^n\in\Lambda_0[X]$ be a poly-

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