44. On a Weak Generalization of the Fundamental Theorem of the Theory of Curves or Hypersurfaces[†]

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0. Introduction. Let us consider the Euclidean space $\boldsymbol{R}^3$ and a surface with an analytic representation ${}^t(f_1, f_2, f_3)=f(x_1, x_2)$. Then, for this, we have first fundamental quantities $K_{ij}(j_x^1(f))=p_i\cdot p_j$ $(1\leq i,\ j\leq 2)$ and second fundamental quantities $L_{ij}(j_x^2(f))=|p_{ij},\ p_1,\ p_2|/\sqrt{K_{11}K_{22}-(K_{12})^2}$ $(1\leq i,\ j\leq 2)$ where the dot means the canonical inner product in $\boldsymbol{R}^3$ and $p_i={}^t(\partial f_1/\partial x_i,\ \partial f_2/\partial x_i,\ \partial f_3/\partial x_i)$, $p_{ij}={}^t(\partial^2 f_1/\partial x_i\partial x_j,\ \partial^2 f_2/\partial x_i\partial x_j,\ \partial^2 f_3/\partial x_i\partial x_j)$. K_{ij} (resp. L_{ij}) is considered as a function on the 1-jet space $J^1(\boldsymbol{R}^2, \boldsymbol{R}^3)$ (resp. the 2-jet space $J^2(\boldsymbol{R}^2, \boldsymbol{R}^3)$). For the above particular f, if we set $\lambda_{ij}(x)=K_{ij}(j_x^1(f))$ and $\eta_{ij}(x)=L_{ij}(j_x^2(f))$, then we get a system of differential equations $P: K_{ij}-\lambda_{ij}=0$ $(1\leq i,\ j\leq 2)$, $L_{ij}-\eta_{ij}=0$ $(1\leq i,\ j\leq 2)$.

Let Γ be the pseudogroup generated by local isometries on the Euclidean space $\boldsymbol{R}^3$. Then the fundamental theorem of the theory of surfaces means that any solution s of P is written by $s=\sigma\circ f$ for some $\sigma\in\Gamma$.

A similar fact holds for curves in $\boldsymbol{R}^3$ with an analytic representation ${}^t(f_1, f_2, f_3)=f(t)$ using the torsion and the curvature of f.

The purpose of this note is to generalize the above stated facts to any local immersion $f: \boldsymbol{R}^n\rightarrow\boldsymbol{R}^m$ $(n<m)$ and any pseudogroup Γ of finite type on $\boldsymbol{R}^m$ in a generic situation for f and Γ. The smoothness is always assumed to be of class C^∞.

1. Statement of the results. Let $J^k(n, m)$ be the space of k-jets of local maps of $\boldsymbol{R}^n$ to $\boldsymbol{R}^m$. If $n<m$, then $\tilde{J}^k(n, m)$ means the space of k-jets of local immersions and if $n\geq m$, $\tilde{J}^k(n, m)$ means the space of k-jets of local submersions. In both cases, $\tilde{J}^k(n, m)$ is open and dense in $J^k(n, m)$.

Let Γ be a pseudogroup on $\boldsymbol{R}^m$. Then Γ is lifted to a pseudogroup $\Gamma_n^{(k)}$ on $\tilde{J}^k(n, m)$ by $\phi^{(k)}(j_x^k(f))=j_x^k(\phi\circ f)$.

A vector field X on $\boldsymbol{R}^m$ is called a Γ-vector field if the local 1-parameter group of local transformations on $\boldsymbol{R}^m$ generated by X is contained in Γ. Let $\mathcal{L}$ denote the sheaf on $\boldsymbol{R}^m$ of germs of Γ-vector fields. Then $\mathcal{L}$ is also lifted to a sheaf $\mathcal{L}_n^{(k)}$ on $\tilde{J}^k(n, m)$. $(\mathcal{L}_n^{(k)})_p$ (resp. $\mathcal{L}_z$) means the stalk of $\mathcal{L}_n^{(k)}$ (resp. $\mathcal{L}$) on $p\in\tilde{J}^k(n, m)$ (resp. $z\in\boldsymbol{R}^m$).

Definition 1.1. A function ϕ on a neighbourhood of a point $p\in\tilde{J}^k(n, m)$ is called a differential invariant of Γ at p if $X\phi=0$ for any $X\in(\mathcal{L}_n^{(k)})_p$.

Let $\{\phi_1, \cdots, \phi_r\}$ be a maximal family of differential invariants of Γ at $j_x^k(f)$ such that the differentials $d\phi_1, \cdots, d\phi_r$ are linearly independent at $j_x^k(f)$

[†] Dedicated to Professor Kenichi SHIRAIWA on his 60th birthday.