43. On Representations of Lie Superalgebras. II

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In this note we introduce a new method of constructing irreducible unitary representations (=IURs) of a classical Lie superalgebra of type A. Then we classify all the irreducible unitary representations of real forms of Lie superalgebra $A(1,0)$. In the previous papers [2], [3], we define unitary representations of Lie superalgebras and introduce a general method of constructing irreducible representations of any simple Lie superalgebras. Moreover we classify and construct all the irreducible (unitary) representations of classical Lie superalgebra $\mathfrak{osp}(1,2)$. Further we gave similar results for real forms of the Lie superalgebra $\mathfrak{sl}(2,1)$ $(=A(1,0))$ exhaustively for the case where the even parts of representations are irreducible. There remains to study the case of non irreducible even parts.

1. New method. We have a Z-gradation $\mathfrak{g}_C=\mathfrak{g}_C^{-1}\oplus\mathfrak{g}_C^0\oplus\mathfrak{g}_C^{+1}$ with $\mathfrak{g}_C^0=\mathfrak{g}_{C,0}$ the even part, of Lie superalgebras $\mathfrak{g}_C=A(n,0)$ compatible with the Z_2-gradation $\mathfrak{g}=\mathfrak{g}_0\oplus\mathfrak{g}_1$ of a real form $\mathfrak{g}$ of $\mathfrak{g}_C$. A new method consists of the following. (i) First we study the weight distributions for IURs (π, V), and see in particular that any IUR is a highest (or lowest) weight representation because of its unitarity (see Proposition 1). (ii) Next we consider induced $\mathfrak{g}_C$-module $\bar{V}(\Lambda)=\mathrm{Ind}_{\mathfrak{p}}^{\mathfrak{g}_C}L(\Lambda)$. Here $\mathfrak{p}=\mathfrak{g}_C^0\oplus\mathfrak{g}_C^{+1}$, and $L(\Lambda)$ is an irreducible highest weight representation of $\mathfrak{g}_{C,0}$ with highest weight Λ considered as $\mathfrak{p}$-module by putting $\mathfrak{g}_C^{+1}$-action as trivial. Any irreducible representation $V(\Lambda)$ of $\mathfrak{g}_C$ with highest weight Λ is a quotient of $\bar{V}(\Lambda)$. (iii) Therefore we should determine the maximal submodule $I(\Lambda)$ of $\bar{V}(\Lambda)$ to get $V(\Lambda)\cong\bar{V}(\Lambda)/I(\Lambda)$.

2. Preliminaries. Denote by $M(p,q;K)$ the set of all matrices of type $p\times q$ with entries in a field K, and by $\mathfrak{g}_C$ the complex algebra $\mathfrak{sl}(n,1)=A(n-1,0)$. Here $\mathfrak{sl}(n,1)$ is realized in $M(n+1,n+1;C)$ as in [4, p. 29]. Let $\mathfrak{h}_C$ be a Cartan subalgebra of $\mathfrak{g}_C$ consisting of diagonal matrices, then $C=\sum_{1\leq i\leq n}E_{i,i}+nE_{n+1,n+1}$ is in $\mathfrak{h}_C$, where $E_{i,j}$ is an element of $M(n+1,n+1;C)$ with components 1 at (i,j) and 0 elsewhere. Real forms $\mathfrak{g}$ of $\mathfrak{g}_C=\mathfrak{sl}(n,1)$ are isomorphic, up to transition to their duals, to one of the following: (a) $\mathfrak{sl}(n,1;R)$; (b) $\mathfrak{su}(n,1;p,1)$ $([n+1/2]\leqq i\leqq n)$. Here $\mathfrak{sl}(n,1;R)=\mathfrak{sl}(n,1)\cap M(n+1,n+1;R)$, and $\mathfrak{su}(n,1;p,1)=\mathfrak{su}(n,1;p,1)_0\oplus\mathfrak{su}(n,1;p,1)_1$ with $\mathfrak{su}(n,1;p,1)_s=\{X\in\mathfrak{sl}(n,1)_s;\ J_pX+(-1)^s\cdot{}^t\bar{X}\bar{J}_p=0\}$ for $s=0,1$, where ${}^tX$ is the transposed matrix of X and $J_p=\mathrm{diag}(1,\cdots,1,-1,\cdots,-1,\sqrt{-1})$ with p-times 1 and $(n-p)$-times -1.