42. On the Initial Value Problem for the Heat Convection Equation of Boussinesq Approximation in a Time-dependent Domain

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Introduction and results. Let K be a compact set in R^m ($m=2$ or 3) with smooth boundary ∂K. Let $\Gamma(t)$ be a simple closed surface in R^3 (or curve in R^2) such that K is contained in the interior of the region surrounded by $\Gamma(t)$. The time-dependent space domain $\Omega(t)$ is a bounded set in R^m whose boundary $\partial\Omega(t)$ consists of two components, i.e.

$$\partial\Omega(t)=\partial K\cup\Gamma(t).$$

Such domains $\Omega(t)$ ($0\leqq t\leqq T$) generate a non-cylindrical domain $\hat{\Omega}=\bigcup_{0\leqq t\leqq T}\Omega(t)\times\{t\}$, where we consider the following initial value problem for the heat convection equation of Boussinesq approximation:

$$(1)\quad \begin{cases} u_t+(u\cdot\nabla)u=-\dfrac{\nabla p}{\rho}+\{1-\alpha(\theta-T_0)\}g+\nu\Delta u & \text{in } \hat{\Omega}, \\ \operatorname{div} u=0 & \text{in } \hat{\Omega}, \\ \theta_t+(u\cdot\nabla)\theta=\kappa\Delta\theta & \text{in } \hat{\Omega}, \end{cases}$$

$$(2)\quad u|_{\partial\Omega(t)}=\beta(x,t),\quad \theta|_{\partial K}=T_0>0,\quad \theta|_{\Gamma(t)}=0 \quad \text{for any } t\in[0,T],$$

$$(3)\quad u|_{t=0}=a,\qquad \theta|_{t=0}=h \qquad \text{in } \Omega(0),$$

where $u=u(x,t)$ is the velocity field, $p=p(x,t)$ is the pressure and $\theta=\theta(x,t)$ is the temperature; ν,κ,α,ρ and $g=g(x)$ are the kinematic viscosity, the thermal conductivity, the coefficient of volume expansion, the density at $\theta=T_0$ and the gravitational vector, respectively. According to Boussinesq approximation, ρ is a fixed constant. The differential operators Δ and ∇ mean those for x variables only. Concerning the Navier-Stokes equation, Fujita-Sauer [1], Ôtani-Yamada [6], Inoue-Wakimoto [2] and H. Morimoto [5] studied the initial value problem or the time periodic problem in some time-dependent domains. As for the stationary problem for the heat convection equation, we refer to, for instance, P.H. Rabinowitz [7] and D.H. Sattinger [8]. We note, as a physical example, the convection of the earth's mantle which may occur in the interior of the earth.

We make some simplifying assumptions on $\beta(x,t)$ and $\Omega(t)$.

(A1) $\beta\equiv 0$. (This may not be physically realistic.)

(A2) There exists an open ball B_1 such that $\overline{\Omega(t)}\subset B_1$.

(A3) $\Gamma(t)$ and ∂K are smooth (say, of class C^3). Also $\Gamma(t)\times\{t\}$ ($0<t<T$) changes smoothly (say, of class C^4) with respect to t. (Namely, the domain $\hat{\Gamma}=\bigcup_{0<t<T}\Gamma(t)\times\{t\}$ has the same properties as those in [1] and [6].)

(A4) $g(x)$ is a bounded and continuous vector function in $R^m\setminus\operatorname{int} K$.