3. On the Cauchy-Kowalewski Theorem for Characteristic Initial Surfaces

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Introduction. We consider the Cauchy problem in the category of holomorphic functions. When the initial surface is non-characteristic, of course we have the well-known theorem of Cauchy-Kowalewski. On the other hand, when it is simply characteristic, Cauchy problem with m initial data is not soluble and that with $m-1$ initial data is not unique (m is the order of the equation), see [4]. Our aim is to show, when the initial surface is characteristic and the multiplicity varies, Cauchy problem with $m-1$ initial data can be uniquely soluble; we give sufficient conditions. The Fuchs type operator with weight $m-1$ will be a particular case.

1. Problem. Let U be a neighborhood of the origin in C^{n+1},

$$P(t, x; \partial_t, \partial_x)=\sum_{s=0}^{m} \sum_{|\alpha|\leqslant s} a_{m-s,\alpha}(t, x)\partial_t^{m-s}\partial_x^{\alpha} \tag{1}$$
$$a_{m-s,\alpha}(t, x) \in \mathcal{O}(U),$$

where $t \in C$, $x=(x_1, \cdots, x_n) \in C^n$, $m \in N$, $\alpha=(\alpha_1, \cdots, \alpha_n)$ multi-index, $|\alpha|=\alpha_1+\cdots+\alpha_n$, $\partial_x^{\alpha}=(\partial/\partial x_1)^{\alpha_1}\cdots(\partial/\partial x_n)^{\alpha_n}$, $\partial_t=\partial/\partial t$, $a(t, x) \in \mathcal{O}(U)$ implies that $a(t, x)$ is defined and holomorphic in U and so is $b(x) \in \mathcal{O}(U_0)$, $U_0=U\cap\{t=0\}$. We denote by P_m the principal part of P and $P_{m(q,\beta)}^{(p,\alpha)}(t, x; \tau, \xi)=\partial_\tau^p\partial_\xi^\alpha\partial_t^q\partial_x^\beta P_m(t, x; \tau, \xi)$, $\tau \in C$, $\xi=(\xi_1, \cdots, \xi_n) \in C^n$.

Assumption A. The hyperplane $t=0$ is characteristic for the operator $P(t, x; \partial_t, \partial_x)$ but not simply characteristic, i.e.

$$P_m(0, x; \tau, 0)\equiv 0, \quad P_m^{(0,\alpha)}(0, 0; \tau, 0)=0 \qquad \text{for all } |\alpha|=1. \tag{2}$$

Under the assumption A, we consider the Cauchy problem

$$(P, m-1): \begin{cases} P(t, x; \partial_t, \partial_x)u=f(t, x) \in \mathcal{O}(U) \\ \partial_t^k u|_{t=0}=g_k(x) \in \mathcal{O}(U_0), \qquad k=0, 1, \cdots, m-2. \end{cases}$$

When there is a neighborhood of the origin V and a unique solution $u \in \mathcal{O}(V)$, we say simply that the Cauchy problem $(P, m-1)$ is uniquely soluble in $\mathcal{O}$.

2. Characteristic coefficients. To state the results, we need to introduce some quantities. First, let

$$\lambda_0=(\partial_t a_{m,0})(0, 0), \qquad \mu=a_{m-1,0}(0, 0). \tag{3}$$

Next, we consider the matrix

$$\left((\partial a_{m-1,e_i}/\partial x_j)(0, 0);\ \begin{matrix} i: 1 \downarrow n \\ j: 1 \rightarrow n \end{matrix}\right) \tag{4}$$

where e_i is the n-dimensional i-th unit vector. Let $\lambda_1, \cdots, \lambda_n$ be the eigenvalues of this matrix. In this paper, we call $\{\lambda_0, \lambda_1, \cdots, \lambda_n, \mu\}$ characteristic