114. *Dual Pairs on Spinors*

Cases of $(\mathrm{C}_m, \mathrm{C}_n)$ *and* $(\mathrm{C}_m^{(1)}, \mathrm{C}_n^{(1)})$

By Kohji HASEGAWA
Department of Mathematics, Nagoya University

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§0. **Introduction.** Weyl's reciprocity theorem says that the symmetric group $\mathfrak{S}_m$ and the general linear group $\mathrm{GL}(n, \boldsymbol{C})$ are mutually commutant (i.e. $(\mathfrak{S}_m, \mathrm{GL}(n, \boldsymbol{C}))$ forms a *dual pair* [3]) on the tensor space $(\boldsymbol{C}^n)^{\otimes m}$. The purpose of this paper is to give the spinor analogues of this theorem: we claim $(\mathfrak{sp}(2m), \mathfrak{sp}(2n))$ forms a dual pair on the $\mathfrak{o}(4mn)$-module $\wedge(\boldsymbol{C}^{2mn})$, and describe its irreducible decomposition as a $\mathfrak{sp}(2m)\oplus\mathfrak{sp}(2n)$-module (Theorem A). The affine Lie algebra pair $(\mathrm{C}_m^{(1)}, \mathrm{C}_n^{(1)})$ also forms a dual pair on $\wedge(\hat{W}_{4mn}^{-})$ (Theorem B). As corollaries we deduce new dualities for branching rules. Details appear in our forthcoming paper [2], where we also construct various dual pairs for all classical Lie algebras, and for their affinizations. Our method is similar to that of [3], which deals with dual pairs on the Shale-Weil modules.

§1. **Finite dimensional case.** **1.1.** After [1] we review the spinor representation of the orthogonal Lie algebra $\mathfrak{o}(2l)=\left\{X\in\mathfrak{gl}(2l)\,\middle|\,{}^tX\begin{bmatrix}0 & \mathbf{1}_l\\ \mathbf{1}_l & 0\end{bmatrix}+\begin{bmatrix}0 & \mathbf{1}_l\\ \mathbf{1}_l & 0\end{bmatrix}X=0\right\}$. Let $C(W_{2l})$ be the Clifford algebra over the vector space $W_{2l}:=V^l\oplus V_l\simeq\boldsymbol{C}^{2l}$, where $V^l:=\oplus_{j=1}^l\boldsymbol{C}\psi^j$ and $V_l:=\oplus_{j=1}^l\boldsymbol{C}\psi_j$, with a symmetric bilinear form $(\ ,\ )$ defined by

$$(\psi^i, \psi_j)=\delta^i{}_j \quad\text{and}\quad (\psi^i, \psi^j)=0=(\psi_i, \psi_j) \quad \text{for } 1\leq i,\ j\leq l.$$

As a $\boldsymbol{C}$-algebra $C(W_{2l})\simeq\mathrm{Mat}(2^l, \boldsymbol{C})$. Its irreducible representation is realized on the exterior algebra $\wedge(V^l)$, with defining 1 the vacuum vector and V^l (resp. V_l) the creation (resp. annihilation) operators. Write $[a, b]$ for $ab-ba$, and the spinor representation s is defined by

$$s:\mathfrak{o}(2l)\ni\begin{bmatrix}E^i{}_j & 0\\ 0 & -E^j{}_i\end{bmatrix}\longmapsto\frac{1}{2}[\psi^i, \psi_j]\in C(W_{2l})\simeq\mathrm{End}\wedge(V^l),$$

$$\begin{bmatrix}0 & E^i{}_j-E^j{}_i\\ 0 & 0\end{bmatrix}\longmapsto\frac{1}{2}[\psi^i, \psi^j],\quad \begin{bmatrix}0 & 0\\ E^i{}_j-E^j{}_i & 0\end{bmatrix}\longmapsto\frac{1}{2}[\psi_i, \psi_j]\qquad (1\leq i, j\leq l).$$

1.2. Now we deal with the dual pair $(\mathfrak{sp}(2m), \mathfrak{sp}(2n))$. Recall that

$$\mathfrak{sp}(2n):=\left\{X\in\mathfrak{gl}(2n)\,\middle|\,{}^tX\begin{bmatrix} & \mathbf{1}_n\\ -\mathbf{1}_n & \end{bmatrix}+\begin{bmatrix} & \mathbf{1}_n\\ -\mathbf{1}_n & \end{bmatrix}X=0\right\}$$
$$=\left\{\begin{bmatrix}A & B\\ C & -{}^tA\end{bmatrix}\,\middle|\,\begin{matrix}A, B, C\in\mathfrak{gl}(n)\\ {}^tB=B,\ {}^tC=C\end{matrix}\right\},$$

and let $l=2mn$. Then there exist two Lie algebra monomorphisms $R:\mathfrak{sp}(2n)\to\mathfrak{o}(2l)$ and $L:\mathfrak{sp}(2m)\to\mathfrak{o}(2l)$ so that $R(\mathfrak{sp}(2n))'=L(\mathfrak{sp}(2m))$ and $L(\mathfrak{sp}(2m))'$