# 111. A Calculus of the Tensor Product of Two Holonomic Systems with Support on Non-singular Plane Curves

By Shinichi TAJIMA
Faculty of General Education, Niigata University

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The aim of this paper is to calculate (in the framework of $\mathcal{D}_X$-Modules) the tensor product of two holonomic systems supported on non-singular plane curves.

§0. **Notation.** Let $X$ be a domain in $C^2$ containing the origin $P=(0,0)$. Let $\mathcal{O}_X$ be the sheaf of germs of holomorphic functions and $\mathcal{D}_X$ the sheaf on X of rings of linear partial differential operators of finite order with holomorphic coefficients. Let $F$ be an analytic plane curve (on $X$) passing through $P$ with a defining equation $f=0$. Let us denote by $\mathcal{H}^1_{[F]}(\mathcal{O}_X)$ the sheaf of algebraic local cohomology with supports in $F$:

$$\mathcal{H}^1_{[F]}(\mathcal{O}_X)=\varinjlim_k \mathcal{E}xt^1_{\mathcal{O}_X}(\mathcal{O}_X/(f)^k,\ \mathcal{O}_X)=\mathcal{O}_X[f^{-1}]/\mathcal{O}_X.$$

Note that the module $\mathcal{H}^1_{[F]}(\mathcal{O}_X)$, which is endowed with a natural structure of left $\mathcal{D}_X$-Module, is a holonomic system.

§1. **Statement of the results.** Let $F$ and $G$ be plane curves meeting properly at a point $P$. We set:

$$\begin{aligned}\mathcal{L}&=\mathcal{H}^1_{[F]}(\mathcal{O}_X)\hat{\otimes}\mathcal{H}^1_{[G]}(\mathcal{O}_X)\\&=\mathcal{D}_{X\times X}\otimes_{p_1^{-1}\mathcal{D}_X\otimes p_2^{-1}\mathcal{D}_X}(p_1^{-1}\mathcal{H}^1_{[F]}(\mathcal{O}_X)\otimes p_2^{-1}\mathcal{H}^1_{[G]}(\mathcal{O}_X)),\end{aligned}$$

where $p_1$ and $p_2$ are the first and the second projections from $X\times X$ to $X$. The following quasi-isomorphism is a special case of a result of Kashiwara [2]:

$$\mathcal{H}^1_{[F]}(\mathcal{O}_X)\overset{L}{\otimes}_{\mathcal{O}_X}\mathcal{H}^1_{[G]}(\mathcal{O}_X)=\mathcal{D}_{X\to X\times X}\overset{L}{\otimes}_{\mathcal{D}_{X\times X}}\mathcal{L}.$$

we have the following

**Theorem 1** (Intersection formula). *Let $F$ and $G$ be non-singular plane curves (on $X$) intersecting properly at $P$. We assume $F\cap G=P$. Then we have the following isomorphisms of $\mathcal{D}_X$-Modules.*

(1) $\mathcal{T}or_k^{\mathcal{D}_{X\times X}}(\mathcal{D}_{X\to X\times X},\mathcal{L})=0$ *for* $k\neq 0$,

(2) $\mathcal{H}^1_{[F]}(\mathcal{O}_X)\otimes_{\mathcal{O}_X}\mathcal{H}^1_{[G]}(\mathcal{O}_X)=\mathcal{D}_{X\to X\times X}\otimes_{\mathcal{D}_{X\times X}}\mathcal{L}=\mathcal{H}^2_{[P]}(\mathcal{O}_X)$,

*where $\mathcal{H}^2_{[P]}(\mathcal{O}_X)$ is the $\mathcal{D}_X$-Module of algebraic local cohomology with supports in $P$.*

**Remark 2.** In the case where $F$ and $G$ being transversal the results above are well known (cf. Sato–Kawai–Kashiwara [3], Schapira [4]).

**Example 3.** Set $X=\{(x,y)\in C^2\}$, $X_1=\{(x_1,y_1)\in C^2\}$, and $X_2=\{(x_2,y_2)\in C^2\}$. $X_1$ and $X_2$ are two copies of $X$. Put $F=\{(x_1,y_1)\mid y_1=0\}$, $G=\{(x_2,y_2)\mid y_2-x_2^2=0\}$. We denote by $\delta(y_1)$ (resp. $\delta(y_2-x_2^2)$) the canonical generator of