109. On Periodic Solutions for the Periodic Quasilinear Ordinary Differential System Containing a Parameter

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1. Introduction. In this paper we deal with the dependence on a parameter λ of T-periodic solutions for the T-periodic quasilinear ordinary differential system:

(1) $$x'=A(t,x,\lambda)x+\lambda F(t,x,\lambda)+f(t).$$

Here A is a real $n\times n$ matrix and F is an $\boldsymbol{R}^n$-valued function. We assume that A and F are defined on $\boldsymbol{R}\times\boldsymbol{R}^n\times[-\lambda_0,+\lambda_0]$, continuous in (t,x,λ) and T-periodic in t, where $\lambda_0>0$. We assume that f is an $\boldsymbol{R}^n$-valued function continuous on $\boldsymbol{R}$ and T-periodic.

We consider the associated T-periodic linear system:

(2) $$x'=B(t)x+f(t),$$

where B is a real $n\times n$ matrix continuous on $\boldsymbol{R}$ and T-periodic.

Hypothesis 1. *For every f continuous on $\boldsymbol{R}$ and T-periodic, there exists one and only one T-periodic solution for* (2).

The qualitative studies of solutions for the periodic quasilinear differential system have been made under Hypothesis 1 (see [1], [2]). When λ is sufficiently small, Cronin [1] has discussed the existence of T-periodic solutions for

(3) $$x'=B(t)x+\lambda F(t,x,\lambda)+f(t)$$

by applying the implicit function theorem. When the Lipschitz conditions are satisfied, Hale [2] has dealt with the continuous dependence on λ of the T-periodic solution for (3) under some additional assumptions.

Theorem 1 in the present paper is the existence theorem of periodic solutions for periodic linear systems which are close to (2) in some sense. Theorem 2 is a strict extension of the standard result (see [1]). Moreover we give an extent that shows how A in (1) is close to B in (2) as well as an extent that shows how small λ is. In Theorem 3 we obtain sufficient conditions for some dependence on λ of periodic solutions for (1). Explicit conditions in Theorem 4 ensure the continuous dependence on λ of the periodic solution for (1).

2. Preliminaries. The symbol $\|\cdot\|$ will denote a norm in $\boldsymbol{R}^n$ and the corresponding norm for $n\times n$ matrices. Let C_T be the space of $\boldsymbol{R}^n$-valued functions continuous on $\boldsymbol{R}$ and T-periodic with the supremum norm. Let $C[0,T]$ be the space of $\boldsymbol{R}^n$-valued functions continuous on $[0,T]$ with the supremum norm $\|\cdot\|_\infty$.