104. The 0-minimal Ideal of the Global of a Combinatorial Completely 0-simple Semigroup

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1. Introduction. Let $S=\mathcal{M}^0(I, M, \{1\}; P)$ be a completely 0-simple combinatorial semigroup where $\{1\}$ is the trivial group and $P=(p_{ji})$ is an $M\times I$-matrix over $\{0,1\}$ (see [1], [2]). Let $\mathcal{P}(S)$ be the global i.e. the power semigroup of S. As proved in [3], $\mathcal{P}(S)$ has a unique 0-minimal ideal $\mathcal{C}(S)$, and $\mathcal{C}(S)\cong\mathcal{M}^0(\mathcal{P}(I), \mathcal{P}(M), \{1\}; \tilde{P})$ where $\tilde{P}=(\tilde{p}_{BA})$, $B\in\mathcal{P}(M)$, $A\in\mathcal{P}(I)$, $\tilde{p}_{BA}=1$ if $p_{ji}\neq 0$ for some $j\in B$, $i\in A$; $\tilde{p}_{BA}=0$ otherwise. According to [4], $\mathcal{P}(S)$ has a unique maximal regular zero-free $\mathcal{J}$-class $\mathcal{T}(S)$ and $\mathcal{T}^0(S)=\mathcal{T}(S)\cup\{0\}$ is completely 0-simple and $\mathcal{T}^0(S)\cong\mathcal{M}^0(\bar{Q}(I), \bar{Q}(M), \{1\}; \bar{P})$ where $\bar{Q}(I)$ is the set of all elements A of $\mathcal{P}(I)$ satisfying; there is $j\in M$ such that $p_{ji}\neq 0$ for all $i\in A$; $\bar{Q}(M)$ is dually defined, $\bar{P}=(\bar{p}_{BA})$ where $\bar{p}_{BA}=1$ if $p_{ji}\neq 0$ for all $j\in B$, $i\in A$; $\bar{p}_{BA}=0$ otherwise. Let S_1 and S_2 be completely 0-simple semigroups, $S_1=\mathcal{M}^0(I_1, M_1, \{1\}; P_1)$, $S_2=\mathcal{M}^0(I_2, M_2, \{1\}; P_2)$. The first author has obtained the following:

Lemma 1. [4] *If $\mathcal{T}^0(S_1)\cong\mathcal{T}^0(S_2)$, then $S_1\cong S_2$.*

In this paper we will use this result to prove that $\mathcal{C}(S_1)\cong\mathcal{C}(S_2)$ implies $S_1\cong S_2$.

2. Definitions. Let $X=(x_{ji})$ be an $M\times I$-matrix (i.e. $j\in M, i\in I$) over $\{0,1\}$. Given X, define $X'=(x'_{ji})$ by $x'_{ji}=1$ if $x_{ji}=0$; $x'_{ji}=0$ if $x_{ji}=1$. Let $X=(x_{ji})$ and $Y=(y_{ji})$ be $M_1\times I_1$- and $M_2\times I_2$-matrices over $\{0,1\}$ respectively. We say X is *equivalent* to Y, denoted by $X\sim Y$, if there are bijections σ: $M_1\to M_2$ and τ: $I_1\to I_2$ such that $x_{ji}=y_{\sigma(j),\tau(i)}$ for all $j\in M$, $i\in I$. If every row and every column of X contains 1, then X is called a *sandwich*. Both X and X' are sandwiches if and only if every row and every column of X contains both 0 and 1. Adjoin a new letter 1 to M and I, namely, let $M^1=M\cup\{1\}$, $1\notin M$; $I^1=I\cup\{1\}$, $1\notin I$. Given an $M\times I$-matrix $X=(x_{ji})$ over $\{0,1\}$, we define an $M^1\times I^1$-matrix $X^1=(x^1_{ji})$ over $\{0,1\}$ as follows: For all $j\in M$, $i\in I$,

$$x^1_{ji}=x_{ji},\quad x^1_{j1}=x^1_{1i}=0,\quad x^1_{11}=1.$$

Then $(X^1)'$ is always a sandwich whether X is so or not. Let $X=(x_{ji})$, $j\in M$, $i\in I$, and for each $B\in\mathcal{P}(M)$, $A\in\mathcal{P}(I)$, define a set $X_{BA}=\{x_{ji}: j\in B, i\in A\}$. Given an $M\times I$-matrix $X=(x_{ji})$ over $\{0,1\}$, we define a $\mathcal{P}(M)\times\mathcal{P}(I)$-matrices $\tilde{X}$ and $\bar{X}$ as follows:

$$\tilde{X}=(\tilde{x}_{BA}),\quad \tilde{x}_{BA}=\begin{cases}1 & \text{if } 1\in X_{BA}\\ 0 & \text{if } X_{BA}=\{0\},\end{cases}$$

$$\bar{X}=(\bar{x}_{BA}),\quad \bar{x}_{BA}=\begin{cases}1 & \text{if } X_{BA}=\{1\}\\ 0 & \text{if } 0\in X_{BA}.\end{cases}$$