10. The Dimension Formula of the Space of Cusp Forms of Weight One for $\Gamma_0(p)$

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1. Introduction and the statement of the results. We fix an odd prime number p throughout this paper. Let $\Gamma=\Gamma_0(p)$ and χ be a Dirichlet character modulo p satisfying $\chi(-1)=-1$. We regard χ as a character of $\Gamma_0(p)$ by $\chi(\sigma)=\chi(d)$ $\left(\sigma=\begin{bmatrix}a, b\\ c, d\end{bmatrix}\right)$. The purpose of this note is to offer the dimension formula of $S_1(\Gamma_0(p),\chi)$, using Selberg's trace formula. Coauthors obtained the results independently, but decided to publish them together. Details will be published elsewhere.

Let S be the upper half-plane and $G=SL_2(\boldsymbol{R})$. Put $\tilde{S}=S\times(\boldsymbol{R}/2\pi\boldsymbol{Z})$. G acts on $\tilde{S}$ as in [4]. Let $M(\lambda)$ denote the space of f in $L^2(\Gamma\backslash\tilde{S},\chi)$ such that

$$\tilde{\Delta}f=\lambda f,\ \frac{\partial}{\partial\phi}f=-if\ \left(\tilde{\Delta}=y^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+\frac{5}{4}\frac{\partial^2}{\partial\phi^2}+y\frac{\partial}{\partial x}\frac{\partial}{\partial\phi}\right).$$

Put $\lambda_0=-3/2$. Let $-3/2>\lambda_1>\lambda_2>\lambda_3>\cdots$ be the set of all discrete spectrums of $\tilde{\Delta}$ in $L_0^2(\Gamma\backslash\tilde{S},\chi)$ such that $M(\lambda_i)\neq\{0\}$. It follows from [3] that $S_1(\Gamma_0(p),\chi)$ is isomorphic to $M(\lambda_0)$. Put $d_i=\dim(M(\lambda_i))$, $\lambda_i=-r_i^2-(3/2)$.

For an integral operator K_δ^* (see below), we can rewrite Selberg's trace formula.

Theorem 1. *For* $\operatorname{Re}(\delta)>0$, *we have*

$$(1)\quad \sum_{i=0}^{\infty}h_\delta(r_i)\cdot d_i=J(Id,\delta)+J(E_2,\delta)+J(E_3,\delta)+J(Hyp,\delta)+J(\infty,\delta)+J(0,\delta)$$

$$J(Id,\delta)=2\pi\text{ volume }(\Gamma\backslash S)=(2/3)\pi^2(p+1),\qquad J(E_2,\delta)=0$$

$$J(E_3,\delta)=\frac{8\pi^2}{3\sqrt{-3}\,\delta}\left\{F\left(1,\frac{\delta}{2},1+\delta;\omega\right)-F\left(1,\frac{\delta}{2},1+\delta;\bar{\omega}\right)\right\}\alpha_p\beta_\chi$$

$$J(Hyp,\delta)=2^{\delta+2}\pi B\left(\frac{1}{2},\frac{1+\delta}{2}\right)z(\delta,\chi)$$

$$J(\infty,\delta)=J(0,\delta)=\log(\pi/2p^{3/2})g_\delta(0)+\frac{\Gamma\left(\frac{1+\delta}{2}\right)\Gamma\left(\frac{3+\delta}{2}\right)}{4\Gamma\left(1+\frac{\delta}{2}\right)^2}h_{1+\delta}(0)$$

$$-\frac{1}{2\pi}\int_{-\infty}^{\infty}\left(h_\delta(r)+h_{1+\delta}(r)\frac{\Gamma\left(\frac{1+\delta}{2}\right)\Gamma\left(\frac{3+\delta}{2}\right)}{\Gamma\left(1+\frac{\delta}{2}\right)^2}\right)\psi(1+ir)dr$$

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