83. On a Codimension 2 Bifurcation of Heteroclinic Orbits

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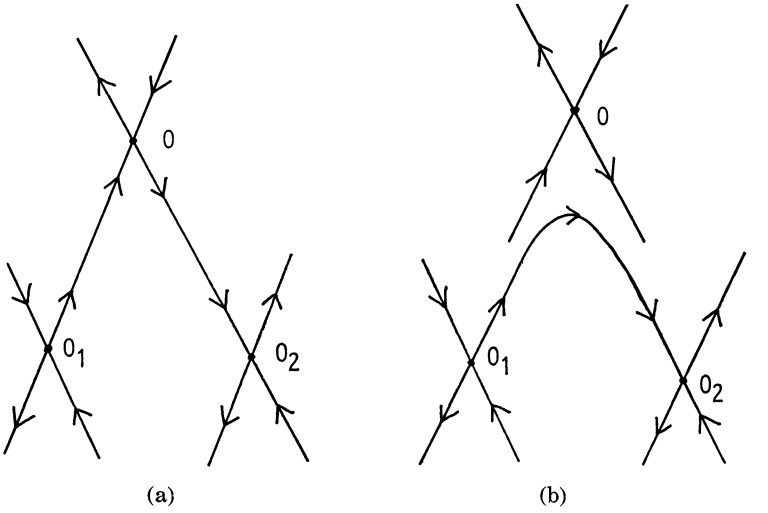
1. Introduction. We consider a bifurcation problem of heteroclinic orbits for a family of ODEs on $\boldsymbol{R}^n$. Suppose there are two heteroclinic orbits, one of which connects saddle points O_1 and O, the origin, and the other connects O and O_2. See Figure (a) below.



Figure

In general, these heteroclinic orbits are broken by perturbations, since they are structurally unstable. We will give below, under some non-degeneracy assumptions, a condition of parameter values for which each heteroclinic orbit persists, and also a condition for which there is a new heteroclinic orbit (Figure (b)) connecting O_1 and O_2 given by joining original heteroclinic orbits near the origin O.

Recent developement of the theory of Melnikiov functions and the exponential dichotomy [3] invoked many works on bifurcations of homoclinic (heteroclinic) orbits, most of which are related to the codimension 2 bifurcation of the vector field singularities ([2], [4] and references therein). From a bifurcation theoretical point of view, it seems more difficult to treat bifurcations of homoclinic and heteroclinic orbits than those of equilibria or periodic orbits, since the formers are global ones.

2. Assume a smooth ODE family $\dot{x}=f(x)+g(x,\mu)$ $(x\in\boldsymbol{R}^n,\ \mu\in\boldsymbol{R}^k)$ with $f(0)=g(0,\mu)=g(x,0)=0$, has three saddle points, $O_1(\mu)$, $O_2(\mu)$ and the origin O. The eigenvalues of the Jacobian matrix at each equilibrium O [resp.