79. *Characterization of the Eigenfunctions in the Singularly Perturbed Domain*

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The eigenvalue problem of the Laplace operator and its relation to the geometrical structure of the domain have been studied by many authors. It is usually the case that the eigenvalue and its eigenfunction vary continuously under the smooth deformation of the domain. But if the deformation of the domain is weak or wild (i.e. the topological type is not conserved or some part of the domain degenerates, etc.), the set of the eigenvalues may not converge to that of the limit domain and some of the eigenfunctions may behave singularly on the perturbed portion of the domain. We deal with a singularly perturbed domain $\Omega(\zeta)=D_1\cup D_2\cup Q(\zeta)$ ($\zeta>0$) where one part $Q(\zeta)$ degenerates to a one-dimensional segment as the parameter $\zeta\to 0$ while $\bar{D}_1\cap\bar{D}_2=\emptyset$ and we characterize the behavior of the eigenfunctions. J. T. Beale [1] has dealt with a domain perturbation similar to the above. He has considered an exterior domain $\mathcal{D}$ of a bounded obstacle with a partially open cavity and characterized the set of the scattering frequencies when the channel to the cavity is sufficiently narrow. Especially in the case of Neumann boundary condition, he has proved that the set of the scattering frequencies is approximated by the union of the scattering frequencies and that of the eigenfrequencies on the limit line segment of the channel $\mathcal{N}$ with the Dirichlet boundary condition on the endpoints of the segment. See [1] for details. By applying the method of [1] to our situation, the set of the eigenvalues $\{\mu_k(\zeta)\}_{k=1}^{\infty}$ of $-\Delta$ for the Neumann boundary condition, is decomposed as follows: $\{\mu_k(\zeta)\}_{k=1}^{\infty}=\{\lambda_k(\zeta)\}_{k=1}^{\infty}\cup\{\omega_k(\zeta)\}_{k=1}^{\infty}$ where $\{\omega_k(\zeta)\}_{k=1}^{\infty}$ approximates the set of the eigenvalues on $D_1\cup D_2$ and $\{\lambda_k(\zeta)\}_{k=1}^{\infty}$ approximates the set of the eigenvalues of $-(d^2/dz^2)$ in the limit line segment for the Dirichlet boundary condition. The purpose of this paper is to present a characterization theorem for the eigenfunctions to $\{\lambda_k(\zeta)\}_{k=1}^{\infty}$ and $\{\omega_k(\zeta)\}_{k=1}^{\infty}$, respectively.

§1. **Formulation.** We specify the singularly perturbed domain $\Omega(\zeta)$ in $\boldsymbol{R}^n$ in the following form,

$$\Omega(\zeta)=D_1\cup D_2\cup Q(\zeta)$$

where $D_i(i=1,2)$ and $Q(\zeta)$ are defined in the following conditions where $x'=(x_2,x_3,\cdots,x_n)\in\boldsymbol{R}^{n-1}$.

(A.1) D_1 and D_2 are bounded domains in $\boldsymbol{R}^n$ (mutually disjoint) with smooth boundary which satisfy the following conditions for some positive constant $\zeta_*>0$.