9. *The Euler Number and Other Arithmetical Invariants for Finite Galois Extensions of Algebraic Number Fields*

By Shin-ichi KATAYAMA
Department of Mathematics, Kyoto University

(Communicated by Shokichi IYANAGA, M. J. A., Feb. 12, 1987)

1. **Introduction.** Let k be an algebraic number field of finite degree over the rational field $\boldsymbol{Q}$. Recently, T. Ono introduced new arithmetical invariants $E(K/k)$ and $E'(K/k)$ for a finite extension K/k. In [5], he obtained a formula between the Euler number $E(K, k)$ and other cohomological invariants for a finite Galois extension K/k. In [2], we obtained a similar formula for $E'(K/k)$. Both proofs use Ono's results on the Tamagawa number of algebraic tori, on which the formulae themselves do not depend. The purpose of this paper is to give a direct proof of these formulae, in response to a problem posed by T. Ono [6]. At the same time, we shall get some relations between $E(K/k)$, $E'(K/k)$ and other arithmetical invariants of K/k (for example, central class number, genus number etc.).

2. Let T be an algebraic torus defined over k, and K- be a Galois splitting field of T. We denote the Galois group $\mathrm{Gal}(K/k)$ by G and the character module $\mathrm{Hom}(T, G_m)$ by $\hat{T}$. $\hat{T}_0$ denotes the integral dual of $\hat{T}$. Let $T(k_A)$, $T(k)$ and $T(k_\mathfrak{p})$ be the k-adelization of T, k-rational points of T and $k_\mathfrak{p}$-rational points of T, where $\mathfrak{p}$ is a place of k. When $\mathfrak{p}$ is finite, we denote the unique maximal compact subgroup of $T(k_\mathfrak{p})$ by $T(O_\mathfrak{p})$. $T(U_k)$ denotes the group

$$\prod_{\mathfrak{p}:\text{finite}} T(O_\mathfrak{p}) \times \prod_{\mathfrak{p}:\text{infinite}} T(k_\mathfrak{p}),$$

where $\mathfrak{p}$ runs over all the places of k. We define the class group of T by putting

$$C(T) = T(k_A)/T(k)\cdot T(U_k).$$

As G-modules, we have

$$T(k_A) \cong (\hat{T}_0 \otimes K_A^\times)^G,$$
$$T(k) \cong (\hat{T}_0 \otimes K^\times)^G,$$
$$T(U_k) \cong (\hat{T}_0 \otimes U_K)^G.$$

Here

$$U_K = \prod_{\mathfrak{P}:\text{finite}} O_\mathfrak{P}^\times \times \prod_{\mathfrak{P}:\text{infinite}} K_\mathfrak{P}^\times,$$

where $\mathfrak{P}$ runs over all the places of K. We note here that $h(T)$, the class number of the torus T, is the order of the group $C(T)$. First, we shall sketch a new direct proof of the equation between $E(K/k)$ and the cohomological invariants of K/k. Consider the following exact sequence of algebraic tori defined over k

$$(1) \qquad 0 \longrightarrow R_{K/k}^{(1)}(G_m) \longrightarrow R_{K/k}(G_m) \xrightarrow{N} G_m \longrightarrow 0.$$