62. *Differentiable Vectors and Analytic Vectors in Completions of Certain Representation Spaces of a Kac-Moody Algebra*

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Let $\mathfrak{g}_k$ be a Kac-Moody algebra over a field k with a symmetrizable generalized Cartan matrix, and $\mathfrak{h}_k$ the Cartan subalgebra of $\mathfrak{g}_k$. Here, we concentrate on the case $k=\boldsymbol{R}$ or $\boldsymbol{C}$, the real or complex number field. We denote $\mathfrak{g}_{\boldsymbol{C}}$ and $\mathfrak{h}_{\boldsymbol{C}}$ simply by $\mathfrak{g}$ and $\mathfrak{h}$ respectively, then $\mathfrak{g}=\boldsymbol{C}\otimes_{\boldsymbol{R}}\mathfrak{g}_{\boldsymbol{R}}$, and $\mathfrak{h}=\boldsymbol{C}\otimes_{\boldsymbol{R}}\mathfrak{h}_{\boldsymbol{R}}$. Let $\mathfrak{k}$ be the unitary form of $\mathfrak{g}$ (cf. [2]). Let Λ be a dominant integral element in $\mathfrak{h}_{\boldsymbol{R}}^*$, and $L(\Lambda)$ the irreducible highest weight module for $\mathfrak{g}$ with highest weight Λ. We denote by $H(\mathrm{ad})$ and $H(\Lambda)$ the completions of $\mathfrak{g}$ and $L(\Lambda)$ with respect to the standard inner products $(\cdot|\cdot)_1$ and $(\cdot|\cdot)_\Lambda$, respectively. In [2], we defined a group K^Λ associated with $\mathfrak{k}$ as a subgroup of the unitary group on the Hilbert space $H(\Lambda)$ generated by the naturally defined exponentials of elements in $\mathfrak{k}$, and then proved that any element in $\mathrm{L}(\Lambda)$ is differentiable and analytic for actions of the exponentials. In this paper, we extend these results so that we can treat the case of adjoint representation as well. Further we show some properties of the exponentials needed to study fine structures of K^Λ.

§ 1. **Basic facts for Kac-Moody algebras.** The notations used here are the same as in [2]. The standard contravariant Hermitian form $(\cdot|\cdot)_0$ on $\mathfrak{g}$ is, unfortunately, not positive definite on $\mathfrak{h}$ in general, though it is always positive definite on each root space $\mathfrak{g}^\alpha$. So, we introduce a new inner product $(\cdot|\cdot)_1$ on $\mathfrak{g}$ as follows: first on $\mathfrak{h}$

$$(h|h')_0\leqq\|h\|_1\|h'\|_1,\quad \lambda(h)\leqq\|\lambda\|_1\|h\|_1 \text{ for } h,\ h'\in\mathfrak{h},\quad \lambda\in\mathfrak{h}^*,$$

where $\|h\|_1=(h|h)_1^{1/2}$, and $\|\cdot\|_1$ on $\mathfrak{h}^*$ is the dual of $\|\cdot\|_1$ on $\mathfrak{h}$. Then extend it to the whole space $\mathfrak{g}$ by

$$(x|x')_1=(x_-|x'_-)_0+(x_0|x'_0)_1+(x_+|x'_+)_0$$

for $x=x_-+x_0+x_+$, $x'=x'_-+x'_0+x'_+\in\mathfrak{g}$ with $x_\pm$, $x'_\pm\in\mathfrak{n}_\pm=\sum_{\alpha\in\Delta_+}\mathfrak{g}^{\pm\alpha}$, and x_0, $x'_0\in\mathfrak{h}$.

Denote by $\underline{\mathfrak{g}}$ the infinite direct product of $\mathfrak{g}^0=\mathfrak{h}$ and of all the root spaces $\mathfrak{g}^\alpha$ $(\alpha\in\Delta)$, and $\underline{L}(\Lambda)$ that of all the weight spaces $L(\Lambda)_\mu$ of $L(\Lambda)$. Each element in $\mathfrak{g}$ acts both on $\underline{\mathfrak{g}}$ and on $\underline{L}(\Lambda)$ naturally. The completions $H(\mathrm{ad})$ and $H(\Lambda)$ of $\mathfrak{g}$ and $L(\Lambda)$ are defined as Hilbert spaces contained in $\underline{\mathfrak{g}}$ and $\underline{L}(\Lambda)$ respectively as:

$$H(\mathrm{ad})=\{(x_\alpha)_\alpha\in\underline{\mathfrak{g}}\,;\,\sum_\alpha\|x_\alpha\|_1^2<+\infty\},\qquad H(\Lambda)=\{(v_\mu)_\mu\in\underline{L}(\Lambda)\,;\,\sum_\mu\|v_\mu\|_\Lambda^2<+\infty\}.$$

§ 2. **Estimates of norms of $\mathfrak{g}$-action.** An element h_0 in $\mathfrak{h}_{\boldsymbol{R}}$ is called