52. On the Existence of Solutions for the Boundary Value Problem of Quasilinear Differential Equations on an Infinite Interval

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1. Introduction. In this paper we deal with the problem of the existence of solutions for the quasilinear differential system with a boundary condition

$$x' = A(t, x)x + F(t, x) \tag{1}$$

$$\mathcal{N}(x) = 0. \tag{2}$$

Let A be a real $n \times n$ matrix continuous on $\boldsymbol{R}^+ \times \boldsymbol{R}^n$, where $\boldsymbol{R}^+ = [0, +\infty)$, and let F be an $\boldsymbol{R}^n$-valued function continuous on $\boldsymbol{R}^+ \times \boldsymbol{R}^n$. We assume that $\mathcal{N}$ is a continuous operator from C_r^{lim} into $\boldsymbol{R}^n$ (not necessarily linear), where $C_r^{\mathrm{lim}} = \{x \in C(\boldsymbol{R}^+) ; \lim_{t \to +\infty} x(t) \text{ exists and } \|x(t)\| \leqq r\}$. We consider the associated linear problem

$$x' = B(t)x \tag{3}$$

$$\mathcal{L}(x) = 0. \tag{4}$$

Let B be a real $n \times n$ matrix continuous on $\boldsymbol{R}^+$ and let $\mathcal{L}$ be a bounded linear operator from C^{lim} into $\boldsymbol{R}^n$, where $C^{\mathrm{lim}} = \{x \in C(\boldsymbol{R}^+) ; \lim_{t \to +\infty} x(t) \text{ exists and is finite}\}$. For example, $\mathcal{L}(x) = Px(0) - Q \lim_{t \to +\infty} x(t)$, where P, Q are known constant $n \times n$ matrices.

Hypothesis H_1. $\int_0^{+\infty} \|B(s)\| ds < +\infty$.

Hypothesis H_2. There exist no solutions for ((3), (4)) except for the zero solution.

In [1], Kartsatos required some qualitative conditions for A in (1) and proved the existence of solutions for ((1), (2)) under the conditions that Hypotheses H_1 and H_2 hold and that A, $\mathcal{N}$ are sufficiently close to B, $\mathcal{L}$ in some sense, respectively. However, the conditions for A are necessary ones if A is sufficiently close to B. We apply a different approach used in [2] and obtain an extention of [1].

2. Preliminaries. The symbol $\|\cdot\|$ will denote a norm in $\boldsymbol{R}^n$ and the corresponding norm for $n \times n$ matrices. Let $C(\boldsymbol{R}^+)$ be the space of $\boldsymbol{R}^n$-valued functions continuous and bounded on $\boldsymbol{R}^+$ with the supremum norm $\|\cdot\|_\infty$. Let $M(\boldsymbol{R}^+)$ be the space of real $n \times n$ matrices continuous and bounded on $\boldsymbol{R}^+$ with the supremum norm $\|A\|_\infty = \sup\{\|A(t)\| ; t \in \boldsymbol{R}^+\}$. We put $\|\mathcal{L}\| = \sup\{\|\mathcal{L}(x)\| ; \|x\|_\infty = 1\}$ and $S_r = \{x \in \boldsymbol{R}^n ; \|x\| \leqq r\}$.

We denote X_B by the fundamental matrix of solutions for (3) such that