24. *Singularities and Cauchy Problem for Fuchsian Hyperbolic Equations*

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In this paper, we shall discuss distribution solutions of the Cauchy problem for Fuchsian hyperbolic equations (in Tahara [2], Bove-Lewis-Parenti [1]), and investigate the propagation of singularities of them by using the notion of wave front sets. The result here is a generalization of results in [1].

1. Fuchsian hyperbolic equations. Let us consider the Cauchy problem:

$$\text{(E)}\qquad \begin{cases} t^k\partial_t^m u+\sum\limits_{\substack{j+|\alpha|\leqq m\\ j<m}} t^{p(j,\alpha)}a_{j,\alpha}(t,x)\partial_t^j\partial_x^\alpha u=f(t,x),\\ \partial_t^i u|_{t=0}=g_i(x), \qquad i=0,1,\cdots,m-k-1, \end{cases}$$

where $(t, x)=(t, x_1, \cdots, x_n)\in[0, T]\times \boldsymbol{R}^n$ $(T>0)$, $m\in \boldsymbol{N}(=\{1, 2, \cdots\})$, $k\in \boldsymbol{Z}_+$ $(=\{0, 1, 2, \cdots\})$, $\alpha=(\alpha_1, \cdots, \alpha_n)\in \boldsymbol{Z}_+^n$, $|\alpha|=\alpha_1+\cdots+\alpha_n$, $p(j, \alpha)\in \boldsymbol{Z}_+ (j+|\alpha|\leqq m$ and $j<m)$, $a_{j,\alpha}(t, x)\in C^\infty([0, T]\times \boldsymbol{R}^n)$ $(j+|\alpha|\leqq m$ and $j<m)$, $\partial_t=\partial/\partial t$, and $\partial_x^\alpha=(\partial/\partial x_1)^{\alpha_1}\cdots(\partial/\partial x_n)^{\alpha_n}$. In addition, we impose the following conditions (A-1)~(A-3) on (E).

(A-1) $0\leqq k\leqq m$.

(A-2) $p(j, \alpha)\in \boldsymbol{Z}_+$ $(j+|\alpha|\leqq m$ and $j<m)$ satisfy

$$\begin{cases} p(j,\alpha)=k+\nu|\alpha|, & \text{when } j+|\alpha|=m \text{ and } j<m,\\ p(j,\alpha)\geqq k-m+j+(\nu+1)|\alpha|, & \text{when } j+|\alpha|<m \end{cases}$$

for some $\nu\in \boldsymbol{Z}_+$.

(A-3) All the roots $\lambda_i(t, x, \xi)$ $(i=1, \cdots, m)$ of

$$\lambda^m+\sum_{\substack{j+|\alpha|=m\\ j<m}} a_{j,\alpha}(t,x)\lambda^j\xi^\alpha=0$$

are *real*, *simple* and *bounded* on $\{(t, x, \xi)\in[0, T]\times \boldsymbol{R}^n\times \boldsymbol{R}^n;\ |\xi|=1\}$.

Then, the equation is one of the most fundamental examples of Fuchsian hyperbolic equations. Therefore, by applying the result in Tahara [2] we can obtain the C^∞ well posedness of (E), that is, the existence, uniqueness and finiteness of propagation speed of solutions in $C^\infty([0, T]\times \boldsymbol{R}^n)$. To prove these results, Tahara [2] used the energy inequality method.

Recently, Bove-Lewis-Parenti [1] has succeeded to construct a right and a left parametrix for the case $\nu=0$, and obtained the existence,

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