81. *Periodicity and Almost Periodicity of Solutions to Free Boundary Problems in Hele-Shaw Flows*

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(Communicated by Kôsaku YOSIDA, M. J. A., Oct. 13, 1986)

This paper is concerned with the asymptotic behavior of solutions to the following problem : given f, g_0, g_1 and u_0, find u such that

$$(1)\qquad \begin{cases} \dfrac{\partial u}{\partial t}-\Delta v=f, & v\in\beta(u) \quad \text{in } (0,\infty)\times\Omega \\ v=g_0 & \text{on } (0,\infty)\times\Gamma_0 \\ \dfrac{\partial v}{\partial n}+p\cdot v=g_1 & \text{on } (0,\infty)\times(\Gamma\setminus\Gamma_0) \\ u(0)=u_0 & \text{in } \Omega, \end{cases}$$

where Ω is a bounded domain in $\boldsymbol{R}^N$ with smooth boundary Γ, Γ_0 is a compact subset of Γ with positive surface measure, p is a positive bounded measurable function on Γ and β is a maximal monotone graph in $\boldsymbol{R}\times\boldsymbol{R}$. In [6] and [7], the global behavior of solutions to (1) is studied in case when β is Lipschitz continuous. This case corresponds to a Stefan problem in a weak sense. But, for instance, in the weak formulations of free boundary problems arising from Hele-Shaw flows and electro-chemical machining processes, β is in general multi-valued (cf. [3, 8, 12, 13, 14, 15]). In [10] and [11], the stability of solutions to general evolution equations generated by time-dependent subdifferentials is studied. But their results do not seem to be directly applicable to our problem. In this paper, we extend a part of the results in [7] to a class of β including the case of Hele-Shaw flows and electro-chemical machining processes.

Let us use the notations : $H=L^2(\Omega)$ with inner product $(\cdot,\cdot)_H$. And put $V=\{z\in H^1(\Omega)\,;\, z=0$ a.e. on $\Gamma_0\}$. Then V becomes a Hilbert space with inner product

$$(z,y)_V=\int_\Omega \nabla z\cdot\nabla y\,\mathrm{d}x+\int_\Gamma p(x)z(x)y(x)\,\mathrm{d}\Gamma \qquad \text{for } z,y\in V.$$

We denote by V^* the dual space of V and regard V^* as a Hilbert space with inner product $(z,y)_*=\langle z,F^{-1}y\rangle_{V^*,V}$ and norm $|z|_*=\langle z,z\rangle_*^{1/2}$ where $\langle\cdot,\cdot\rangle_{V^*,V}$ is the duality between V^* and V and F is the duality mapping from V onto V^*.

Definition 1. For given constants $a>0$ and $b\geqq 0$, let $B(a,b)$ be the set of all maximal monotone graph β in $\boldsymbol{R}\times\boldsymbol{R}$ such that $\beta=\partial\hat\beta$ for some $\hat\beta:\boldsymbol{R}\to\boldsymbol{R}\cup\{\infty\}$ proper l.s.c. (lower-semicontinuous) convex function with $\hat\beta(0)=0$ and $\hat\beta(r)\geqq a|r|^2-b$ for all $r\in\boldsymbol{R}$.