63. *A Characterization of Heegaard Diagrams for the 3-Sphere**)

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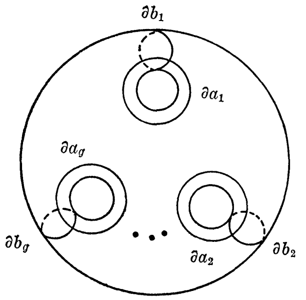
1. Introduction. In [7], Waldhausen showed the uniqueness of Heegaard splittings of genus g for the 3-sphere S^3. However, there are infinitely many Heegaard diagrams of each genus g (>1) for S^3 which look like quite different. No algorithm to distinguish those for S^3 among those for other 3-manifolds has been found in case of $g>2$ until now. There seems to be no good characterization of those for S^3 even if we accept non-algorithmic one. One of the essential difficulties comes from the fact that choice of meridian disks is free in both sides of the splittings. In this paper, we show a characterization (Theorem 2) which is not algorithmic but reduces the problem to the case of free choice of meridian disks in only one side.

To investigate Heegaard diagrams, their associated presentations of the fundamental groups have good information. Another result is a reduction (Theorem 3) of the cyclically reduced presentations associated with Heegaard diagrams for S^3 to those of the special type (S^3; ∂T_g, A, B, $\boldsymbol{a}$, $\boldsymbol{b}'$) (see the next section) where free choice of meridian disks is only in the side B as $\boldsymbol{b}'$. As an application of these results, we shall show a practical algorithmic characterization in the case of genus 2 in [5].

2. Statement of results. We follow [1] or [2] for the precise definitions of Heegaard splitting, diagram and sewing for a 3-manifold. Let T_g be a solid torus of genus g i.e., a 3-ball with g 1-handles. Assume that $S^3=R^3\cup\{\infty\}\supset R^3$ and T_g is embedded in R^3 as shown in Fig. 1.



$R^3\supset B=T_g$

Fig. 1

*) A part of this paper was done while the author was visiting at University of Toronto.