36. *Some Applications of the Generalized Libera Integral Operator*

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Summary. The object of the present paper is to prove several interesting characterization theorems involving the generalized Libera integral operator $\mathcal{J}_c$ and a general class $\mathcal{C}(\alpha, \beta)$ of close-to-convex functions in the unit disk. An application of the integral operator $\mathcal{J}_c$ to a class of generalized hypergeometric functions is also considered.

1. Introduction. Let $\mathcal{A}$ denote the class of functions of the form

$$f(z)=z+\sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the unit disk $\mathcal{U}=\{z:|z|<1\}$. Also let $\mathcal{S}$ denote the class of all functions in $\mathcal{A}$ which are univalent in the unit disk $\mathcal{U}$. Then a function $g(z) \in \mathcal{S}$ is said to be starlike of order α if and only if

$$\mathrm{Re}\left\{\frac{zg'(z)}{g(z)}\right\}>\alpha \tag{1.2}$$

for some α $(0\leqq\alpha<1)$ and for all $z \in \mathcal{U}$. We denote by $\mathcal{S}^*(\alpha)$ the class of all functions in $\mathcal{S}$ which are starlike of order α. Note that

$$\mathcal{S}^*(\alpha)\subseteq\mathcal{S}^*(0)\equiv\mathcal{S}^*\subset\mathcal{S} \qquad (0\leqq\alpha<1). \tag{1.3}$$

Throughout this paper, it should be understood that functions such as $zg'(z)/g(z)$, which have removable singularities at $z=0$, have had these singularities removed in statements like (1.2).

The class $\mathcal{S}^*(\alpha)$ was introduced by Robertson [8], and was studied subsequently by Schild [9], MacGregor [5], Pinchuk [7], Jack [2], and others.

A function $f(z) \in \mathcal{A}$ is said to be in the class $\mathcal{C}(\alpha, \beta)$ if there is a starlike function $g(z)$ of order α such that

$$\mathrm{Re}\left\{\frac{zf'(z)}{g(z)}\right\}>\beta \tag{1.4}$$

for some β $(0\leqq\beta<1)$ and for all $z \in \mathcal{U}$. It follows from (1.4) that

$$\mathcal{C}(\alpha, \beta)\subseteq\mathcal{C}(\alpha, \gamma) \qquad (0\leqq\gamma\leqq\beta<1). \tag{1.5}$$

In particular, $\mathcal{C}(0, 0)$ is the familiar class of close-to-convex functions, and $\mathcal{C}(0, \beta)$ is an important subclass of close-to-convex functions. Thus $\mathcal{C}(\alpha, \beta)$ provides an interesting generalization of the class of close-to-convex functions.

In the present paper we make use of the generalized Libera integral operator $\mathcal{J}_c$, defined by Equation (2.1) below, with a view to proving several

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