26. *On Some Properties of Set-dynamical Systems*

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1. Introduction. In [2], the author investigated self-similar sets, including classical singular curves like Peano's and Koch's, as the invariant sets under several contraction mappings.

In this paper, we shall treat such a peculiar set as the fixed point of a certain set-dynamical system.

Let X be a complete metric space with a metric d. The power set 2^X of all subsets of X forms a partially ordered set under set-inclusion in a natural way, that is, $x \leq y$ means x is a subset of y. Moreover, 2^X is a complete lattice with operations join "+" (set-union) and meet " · " (set-intersection). Let $C(X)$ be a subcollection of 2^X of all non-empty compact subsets of X, which is itself a partially ordered set under the same inclusion relation. Since $C(X) \not\ni \phi$ (empty set), $C(X)$ is not a lattice but a join-semilattice with the binary relation "+".

It is known that $C(X)$ is a complete metric space equipped with the Hausdorff metric:

$$d_H(x, y) = \max(\inf\{\varepsilon > 0,\ N_\varepsilon(x) \geq y\},\ \inf\{\varepsilon > 0,\ N_\varepsilon(y) \geq x\})$$

where $N_\varepsilon(x) \in 2^X$ is an ε-neighbourhood of the set x. Moreover, if X is compact, $C(X)$ becomes also a compact metric space [4]. Note that the mapping $i: X \to C(X)$, which maps p into $\{p\}$, is an isometry.

2. Induced mappings. A mapping $F: C(X) \to C(X)$ is said to be order-preserving provided that $x \leq y$ implies $F(x) \leq F(y)$; a join-endomorphism provided that $F(x+y) = F(x) + F(y)$ for all $x, y \in C(X)$. Let $\mathcal{F}$ consist of all continuous, order-preserving join-endomorphisms defined on $C(X)$; and let $F \leq G$ mean that $F(x) \leq G(x)$ for every $x \in C(X)$. Then $\mathcal{F}$ becomes a join-semilattice with operation "+", that is, $(F+G)(x)$ means $F(x) + G(x)$ for every $x \in C(X)$.

Now let $f: X \to X$ be a continuous mapping. Since the image of $x \in C(X)$ under f is plainly compact, we can define the *induced mapping* $f^*: C(X) \to C(X)$ in a natural way. Note that $(f \circ g)^* = f^* \circ g^*$ for any continuous self-mappings f, g. It is obvious that any induced mapping is contained in $\mathcal{F}$.

A self-mapping h defined on a metric space (E, δ) is said to satisfy the *condition* ψ provided that

$$\delta(h(x), h(y)) \leq \psi(\delta(x, y)) \qquad \text{for every } x, y \in E,$$

where $\psi(t)$ is a non-decreasing right-continuous real-valued function defined on $[0, \infty)$ satisfying $\psi(0) = 0$. Then we have: