19. *Infinitely Many Periodic Solutions for the Equation:* $u_{tt}-u_{xx}\pm|u|^{s-1}u=f(x,t)$

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1. Introduction. In this article we shall study the nonlinear wave equation:

(1)$_\pm$ $\quad v_{tt}-v_{xx}\pm|v|^{s-1}v=f(x,t), \qquad (x,t)\in(0,\pi)\times\boldsymbol{R},$

(2) $\quad v(0,t)=v(\pi,t)=0, \qquad t\in\boldsymbol{R},$

(3) $\quad v(x,t+2\pi)=v(x,t), \qquad (x,t)\in(0,\pi)\times\boldsymbol{R},$

where $s>1$ is a constant and $f(x,t)$ is a 2π-periodic function of t.

Our main result is as follows:

Theorem. *Assume that* $1<s<1+\sqrt{2}$ *and* $f(x,t)\in L^q_{\mathrm{loc}}([0,\pi]\times\boldsymbol{R})$ $(q=1/s+1)$ *is a* 2π*-periodic function of* t. *Then* (1)$_\pm$–(3) *possessess an unbounded sequence of weak solutions in* $L^{s+1}_{\mathrm{loc}}([0,\pi]\times\boldsymbol{R})$.

To prove our theorem, we convert the problem to a simpler one by a Legendre transformation which is used in H. Brézis, J. M. Coron and L. Nirenberg [2], that is, we use the dual variational formulation for (1)$_\pm$–(3). Next we use a perturbation result of P. H. Rabinowitz [3] asserting the existence of infinitely many critical points of perturbed symmetric functionals.

After completing this work, the author knew announcement of the result of J. P. Ollivry [6]. His result is analogous to ours for (1)$_+$–(3) but under the following conditions:

$$1<s<2 \quad \text{and} \quad f(x,t)\in E \quad (\text{see } (4)).$$

Our result obviously contains his result. Moreover our growth restriction $1<s<1+\sqrt{2}$ coincides with the condition which ensures the existence of an unbounded sequence of solutions of the semilinear elliptic equation:

$$-\Delta u=|u|^{s-1}u+f(x), \qquad x\in\Omega,$$
$$u=0, \qquad x\in\partial\Omega,$$

where $\Omega\subset\boldsymbol{R}^2$ is a smooth bounded domain (see P. H. Rabinowitz [3]).

2. Outline of the proof of Theorem. We shall only give outline of proof. Details will be published elsewhere.

We shall deal with the case (1)$_+$–(3) (the argument is essentially the same for the case (1)$_-$–(3)).

Let $\Omega=(0,\pi)\times(0,2\pi)$.

We shall consider the operator $Au=u_{tt}-u_{xx}$ acting on functions in $L^1(\Omega)$ satisfying (2), (3). Denote by N the kernel of A. Consider the space

(4) $\quad E=\left\{u\in L^q(\Omega)\,;\int_\Omega u\phi=0 \text{ for all } \phi\in N\cap L^{s+1}(\Omega)\right\}$

with L^q norm $\|\cdot\|_q$.