17. *Cauchy Problems for Fuchsian Hyperbolic Equations in Spaces of Functions of Gevrey Classes*

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(Communicated by Kôsaku YOSIDA, M. J. A., March 12, 1985)

In this paper, we deal with the Cauchy problem for Fuchsian hyperbolic equations with Gevrey coefficients, and establish the well posedness of the problem in spaces of functions of Gevrey classes.

1. Problem. Let us consider the Cauchy problem:

$$(\mathrm{P}) \qquad \begin{cases} t^k \partial_t^m u + \sum_{\substack{j+|\alpha|\leqq m \\ j<m}} t^{p(j,\alpha)} a_{j,\alpha}(t,x) \partial_t^j \partial_x^\alpha u = f(t,x), \\ \partial_t^i u|_{t=0} = u_i(x), \qquad i=0,1,\cdots,m-k-1, \end{cases}$$

where $(t,x)=(t,x_1,\cdots,x_n)\in[0,T]\times \boldsymbol{R}^n$ $(T>0)$, $m\in \boldsymbol{N}$ $(=\{1,2,\cdots\})$, $k\in \boldsymbol{Z}_+$ $(=\{0,1,2,\cdots\})$, $\alpha=(\alpha_1,\cdots,\alpha_n)\in \boldsymbol{Z}_+^n$, $|\alpha|=\alpha_1+\cdots+\alpha_n$, $p(j,\alpha)\in \boldsymbol{Z}_+$ $(j+|\alpha|\leqq m$ and $j<m)$, $a_{j,\alpha}(t,x)\in C^\infty([0,T]\times \boldsymbol{R}^n)$ $(j+|\alpha|\leqq m$ and $j<m)$, $\partial_t=\partial/\partial t$, and $\partial_x^\alpha=(\partial/\partial x_1)^{\alpha_1}\cdots(\partial/\partial x_n)^{\alpha_n}$. Assume the following condition:

(A-1) $0\leqq k\leqq m$.

(A-2) $p(j,\alpha)\in \boldsymbol{Z}_+$ $(j+|\alpha|\leqq m$ and $j<m)$ satisfy

$$\begin{cases} p(j,\alpha)=k+\langle\nu,\alpha\rangle, & \text{when } j+|\alpha|=m \text{ and } j<m, \\ p(j,\alpha)>k-m+j, & \text{when } j+|\alpha|<m \text{ and } |\alpha|>0, \\ p(j,\alpha)\geqq k-m+j, & \text{when } j+|\alpha|<m \text{ and } |\alpha|=0 \end{cases}$$

for some $\nu=(\nu_1,\cdots,\nu_n)\in \boldsymbol{Q}^n$ such that $\nu_i\geqq 0$ $(i=1,\cdots,n)$, where $\langle\nu,\alpha\rangle=\nu_1\alpha_1+\cdots+\nu_n\alpha_n$.

(A-3) All the roots λ_i (t,x,ξ) $(i=1,\cdots,m)$ of

$$\lambda^m + \sum_{\substack{j+|\alpha|=m \\ j<m}} a_{j,\alpha}(t,x)\lambda^j \xi^\alpha = 0$$

are *real*, *simple* and *bounded* on $\{(t,x,\xi)\in[0,T]\times \boldsymbol{R}^n\times \boldsymbol{R}^n\,;|\xi|=1\}$.

Then, the equation is one of the most fundamental examples of Fuchsian hyperbolic equations. The characteristic exponents $\rho=0,1,\cdots,m-k-1$, $\rho_1(x),\cdots,\rho_k(x)$ are defined by the roots of

$$0=\rho(\rho-1)\cdots(\rho-m+1)+a_{m-1}(x)\rho(\rho-1)\cdots(\rho-m+2) + \cdots + a_{m-k}(x)\rho(\rho-1)\cdots(\rho-m+k+1),$$

where $a_j(x)=(t^{p(j,(0,\cdots,0))-k+m-j}a_{j,(0,\cdots,0)}(t,x))|_{t=0}$ $(j<m)$.

2. Well posedness in $C^\infty([0,T],\mathcal{E}(\boldsymbol{R}^n))$. Let $\mathcal{E}(\boldsymbol{R}^n)$ be the Schwartz space on $\boldsymbol{R}^n$ and let $C^\infty([0,T],\mathcal{E}(\boldsymbol{R}^n))$ be the space of all C^∞ functions on $[0,T]$ with values in $\mathcal{E}(\boldsymbol{R}^n)$. Then, by applying the result in Tahara [6] we have

Theorem 1. *Assume that* (A-1)$\sim$(A-3) *and the condition*:

(T) $\quad p(j,\alpha)\geqq k-m+j+\langle\nu,\alpha\rangle+|\alpha|$, $\quad$ *when* $j+|\alpha|<m$ *and* $|\alpha|>0$

hold, and that $\rho_1(x),\cdots,\rho_k(x)\notin\{\lambda\in \boldsymbol{Z}\,;\lambda\geqq m-k\}$ *for any* $x\in \boldsymbol{R}^n$. *Then, for*