87. A Note on the Spaces of Self Homotopy Equivalences for Fibre Spaces

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1. **Introduction.** Throughout this note, we shall work within the category of compactly generated Hausdorff spaces which will be simply called *spaces*. Let X and Y be spaces with base points x_0 and y_0 respectively. We denote by map(X, Y) and map$_0(X, Y)$ the space of maps of X to Y and the space of maps of (X, x_0) to (Y, y_0) respectively. Moreover, when k is a map of X to Y, we denote by map$(X, Y; k)$ the path component of k in map(X, Y), and map$_0(X, Y; k)$ is defined similarly. A CW complex means a connected CW complex with non-degenerate base point. Let X be a CW complex with base point x_0, $G(X)$ the space of self homotopy equivalences of X and $G_0(X)$ the space of self homotopy equivalences of (X, x_0). In previous papers [5], [6], [7] we studied $G_0(X)$ when $X=E$ is a fibre space of a fibration: $F\xrightarrow{i}E\xrightarrow{p}B$. This paper is also concerned with $G_0(X)$ for a fibre space X.

2. **Main results.** We quote the following two theorems [5, 6].

Theorem A. *Let E and B be CW complexes and $p: E\to B$ a fibration with fibre F. Let $n>1$ be a given integer. If F is $(n-1)$-connected and $\pi_i(B)=0$ for every $i\geqq n$, then we have the following fibration:*

$$\mathcal{G}(E \bmod F)\longrightarrow G_0(E)\xrightarrow{\rho}G_0(B)\times G_0(F),$$

where $\mathcal{G}(E \bmod F)$ is the space of self fibre homotopy equivalences of E leaving the fibre F fixed.

Theorem B. *Under the same hypothesis as above, the image of ρ: $G_0(E)\to G_0(B)\times G_0(F)$ is just the union of the path components in $G_0(B)\times G_0(F)$ each of which contains (g, h) satisfying*

$$[\chi_\infty(h)]\circ[k]=[k]\circ[g],$$

where $\chi_\infty(h)$ is a self map of (B_∞, b_∞) and $k: (B, b_0)\to(B_\infty, b_\infty)$ is a classifying map in Allaud's sense for the fibration: $F\xrightarrow{i}E\xrightarrow{p}B$.

Let $\varepsilon(X)$ denote the group $\pi_0(G_0(X))$ for a CW complex X and let R be a subgroup of $\varepsilon(B)\times\varepsilon(F)$ consisting of the elements $([g], [h])$ satisfying $[\chi_\infty(h)]\circ[k]=[k]\circ[g]$. Then our main result is the following

Theorem 1. *Let E and B be CW complexes and $F\xrightarrow{i}E\xrightarrow{p}B=K(\pi, n)$ a fibration classified by a map $k: (B, b_0)\to(B_\infty, b_\infty)$ in Allaud's sense. Let $n>1$ be a given integer. If F is n-connected and $\pi_j(F)=0$ for every $j\geqq 2n$, then we have the following fibration:*